

Lecture Tue.3

Intro to the EPW code and recent developments

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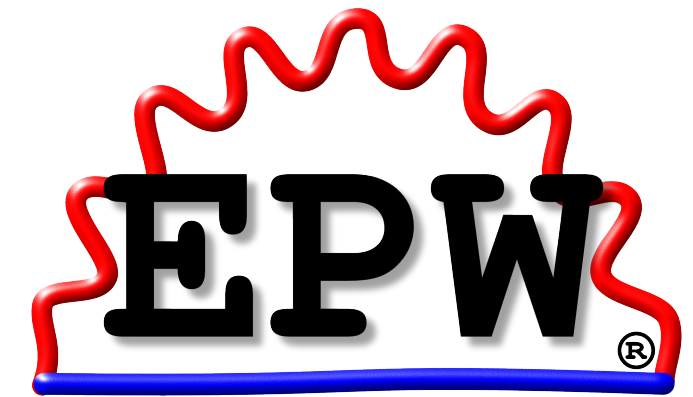
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Lecture Summary

- Brief introduction to EPW
- Computational flow
- Recent developments
- Future plans

Brief introduction to EPW [<https://epw-code.org>]



- Open-source Fortran code to calculate materials properties related to the electron-phonon interaction
 - Based on density-functional perturbation theory and maximally localized Wannier functions.
- One of core modules of Quantum ESPRESSO since 2016

Functionalities of EPW

- Interpolate electron-phonon matrix element on fine momentum grids
- Compute electron and phonon linewidths, scattering rates & lifetimes
- Electron and phonon spectral functions
- Electron-phonon coupling strength
- Superconducting properties
- Transport properties

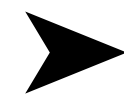
Features of EPW

- Support LDA/GGA functionals
- MPI parallelization:
 - over \mathbf{k} points and bands on coarse grids
 - over \mathbf{k} points on fine grids
- Support spin-orbit coupling
- Support time-reversal symmetry
- Interpolate polar divergence correctly
- Integrated into QE and rely on Wannier90
- Have a test-farm for stability and portability of the code

Computational flow

epw.f90

```
CALL epw_readin()  
..  
IF (wannierize) CALL wann_run()  
CALL elphon_shuffle_wrap()  
>> CALL elphel2_shuffle  
> CALL ephwann_shuffle  
..
```



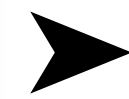
Read input keywords,
read wfns and misc. data from pw.x

Computational flow

epw.f90

```
CALL epw_readin()
```

```
..  
IF (wannierize) CALL wann_run()  
CALL elphon_shuffle_wrap()  
>> CALL elphel2_shuffle  
> CALL ephwann_shuffle  
..
```



Generate inputs to W90 and Wannierization

Computational flow

epw.f90

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CALL epw_readin()  
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IF (wannierize) CALL wann_run()  
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```

➤ Calculated e-ph vertex on coarse grids

Computational flow

epw.f90

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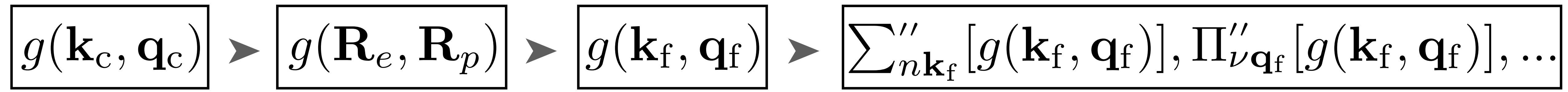
➤ Bloch to Wannier and Wannier to Bloch, + ...

Computational flow:

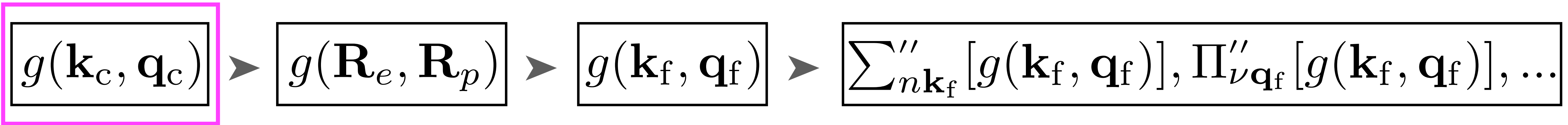
Wannierization

- Call W90 as a library (run serially)
- Generate internally inputs (amn, mmn, etc.) to W90
- Generate directly real-space Wannier orbitals [.cube]
- Main input keywords of W90 ➤ inputs of EPW [Tue.4 Hands-on]
- Others can also be passed to W90 [Tue.4 Hands-on]

Computational flow: e-ph evaluation



Computational flow: e-ph evaluation



Wave functions from pw.x (NSCF)

$$g_{mn\nu}(\mathbf{k}_c, \mathbf{q}_c) = \langle u_{m\mathbf{k}_c+\mathbf{q}_c} | \Delta_{\mathbf{q}_c\nu} v^{\text{KS}} | u_{n\mathbf{k}_c} \rangle_{\text{uc}}$$

Perturbing potentials
(In patterns basis)
from ph.x

$$\Delta_{\mathbf{q}_c\nu} v^{\text{KS}} = \sum_{\kappa\alpha} \sqrt{\frac{\hbar}{2M_\kappa\omega_{\mathbf{q}_c\nu}}} e_{\kappa\alpha,\nu}(\mathbf{q}_c) \partial_{\kappa\alpha,\mathbf{q}_c} v^{\text{KS}}$$

Calculated from
dynamical matrices from ph.x

, where $\partial_{\kappa\alpha,\mathbf{q}_c} v^{\text{KS}} = e^{-i\mathbf{q}_c \cdot \mathbf{r}} \sum_p e^{i\mathbf{q}_c \cdot \mathbf{R}_p} \frac{\partial V^{\text{KS}}(\mathbf{r})}{\partial \tau_{\kappa\alpha p}}$

Technicalities:

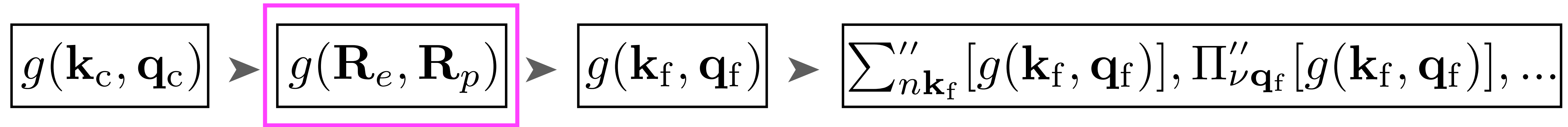
Difference from ph.x

ph.x explicitly calculates the states at \mathbf{k} and $\mathbf{k}+\mathbf{q}$.

EPW folds the states at $\mathbf{k}+\mathbf{q}$ by using the periodic gauge condition

$$u_{n,\mathbf{k}+\mathbf{G}}(\mathbf{r}) = e^{-i\mathbf{G}\cdot\mathbf{r}} u_{n\mathbf{k}}(\mathbf{r})$$

Computational flow: e-ph evaluation



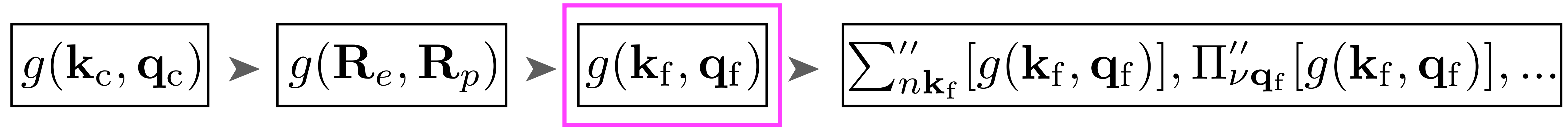
$$g_{mn\kappa\alpha}(\mathbf{R}_p, \mathbf{R}_{p'}) = \langle w_{m0}(\mathbf{R}) | \frac{\partial V^{\text{KS}}}{\partial \tau_{\kappa\alpha}} (\mathbf{r} - \mathbf{R}_{p'}) | w_{m0}(\mathbf{R} - \mathbf{R}_p) \rangle_{\text{sc}}$$

[Phys. Rev. B **76**, 165108 (2007), Rev. Mod. Phys. **89**, 1 (2017)]

Use of minimal-distance replica selection
for Wannier interpolation

[J. Phys. Cond. Matt. 32, 165902 (2020), arXiv:2105.04192 (2021)]

Computational flow: e-ph evaluation

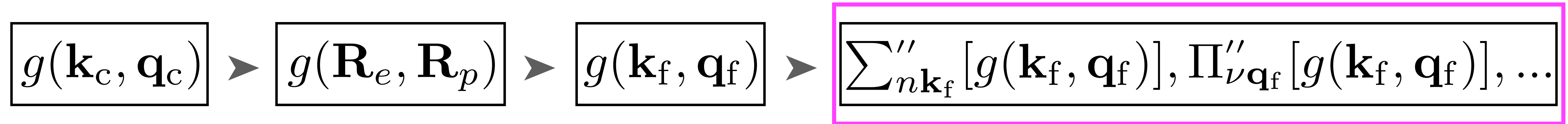


$$\begin{aligned}
 g_{mn\nu}(\mathbf{k}_f, \mathbf{q}_f) &= \sqrt{\frac{\hbar}{2M_{\kappa}\omega_{\mathbf{q}_f\nu}}} \sum_{pp'} e^{i(\mathbf{k}_f \cdot \mathbf{R}_p + \mathbf{q}_f \cdot \mathbf{R}_{p'})} \\
 &\times [U_{\mathbf{k}_f + \mathbf{q}_f} g(\mathbf{R}_p, \mathbf{R}_{p'}) \cdot e_{\kappa\alpha, \nu}(\mathbf{q}_f) U_{\mathbf{k}_f}^{\dagger}]_{mn}
 \end{aligned}$$

[Phys. Rev. B **76**, 165108 (2007), Rev. Mod. Phys. **89**, 1 (2017)]

Use of crystal symmetry (w.r.t \mathbf{k}) and
Fermi window

Computational flow: e-ph evaluation



$$\begin{aligned}
 \Sigma''_{n\mathbf{k}_f}(\omega, T) = & \pi \sum_{m\nu} \int_{\text{BZ}} \frac{d\mathbf{q}_f}{\Omega_{\text{BZ}}} |g_{mn\nu}(\mathbf{k}_f, \mathbf{q}_f)|^2 \\
 & \{ [n_{\mathbf{q}_f\nu}(T) + f_{m\mathbf{k}_f+\mathbf{q}_f}(T)] \delta(\omega - (\varepsilon_{m\mathbf{k}_f+\mathbf{q}_f} - \varepsilon_F) + \omega_{\mathbf{q}_f\nu}) \\
 & + [n_{\mathbf{q}_f\nu}(T) + 1 - f_{m\mathbf{k}_f+\mathbf{q}_f}(T)] \delta(\omega - (\varepsilon_{m\mathbf{k}_f+\mathbf{q}_f} - \varepsilon_F) - \omega_{\mathbf{q}_f\nu}) \}
 \end{aligned}$$

[Comput. Phys. Commun. **209**, 116 (2016), Rev. Mod. Phys. **89**, 1 (2017)]

Technicalities:

e-ph evaluation on coarse grids

○ Use of the periodic gauge condition for folding $u_{n,\mathbf{k}+\mathbf{G}}(\mathbf{r}) = e^{-i\mathbf{G}\cdot\mathbf{r}}u_{n\mathbf{k}}(\mathbf{r})$

○ Use of crystal symmetry

$$\begin{aligned}g_{mn\nu}(\mathbf{k}, S\mathbf{q}) &= \langle \psi_{m,\mathbf{k}+S\mathbf{q}}(\mathbf{r}) | \partial_{\nu,S\mathbf{q}} V(\mathbf{r}) | \psi_{n,\mathbf{k}}(\mathbf{r}) \rangle \\ &= \langle \psi_{m,\mathbf{k}+S\mathbf{q}}(\mathbf{r}) | \partial_{\nu\mathbf{q}} V(\{S|\mathbf{v}\}^{-1}\mathbf{r}) | \psi_{n,\mathbf{k}}(\mathbf{r}) \rangle \\ &= \langle \psi_{m,\mathbf{k}+S\mathbf{q}}(\{S|\mathbf{v}\}\mathbf{r}) | \partial_{\nu\mathbf{q}} V(\mathbf{r}) | \psi_{n,\mathbf{k}}(\{S|\mathbf{v}\}\mathbf{r}) \rangle\end{aligned}$$

○ Use of time-reversal symmetry

(only when the small group of \mathbf{q} doesn't include $\mathbf{q} \leftrightarrow -\mathbf{q}$ operation)

$$g_{mn\nu}(\mathbf{k}, -S\mathbf{q}) = \langle \psi_{m,\mathbf{k}-S\mathbf{q}}(\{S|\mathbf{v}\}\mathbf{r}) | \left(\partial_{\nu\mathbf{q}} V(\mathbf{r}) \right)^* | \psi_{n,\mathbf{k}}(\{S|\mathbf{v}\}\mathbf{r}) \rangle$$

[Phys. Rev. B **76**, 165108 (2007), Comput. Phys. Commun. **209**, 116 (2016)]

General design principles

- Improve the computational workflow
- Reduce the computational size
- Optimize the computation

Improvement on workflow

- Divide EPW into two separate codes
- Incorporate NSCF step into EPW
- Start from irreducible coarse grids.
- Other simplifications

Efficient calculation of e-ph vertex

- e-ph vertex is a main ingredient to calculate electron-phonon related quantities

$$g_{mn\nu}(\mathbf{k}, \mathbf{q}) = \langle \psi_{m, \mathbf{k}+\mathbf{q}}(\mathbf{r}) | \partial_{\nu, \mathbf{q}} V(\mathbf{r}) | \psi_{n, \mathbf{k}}(\mathbf{r}) \rangle$$

- A large portion of time spent on its evaluation in EPW
- Calculated in plane-waves basis on coarse grids and in reduced basis on dense grids after Wannier interpolation
- Gauges of wave functions are arbitrary, but consistent ones should be used throughout all calculations
- On coarse grids, calculated for all pairs of states at \mathbf{k} and $\mathbf{k}+\mathbf{q}$ $N_b^2 \times N_k \times N_q \times N_\nu$ (evaluations)
(number of bands N_b , \mathbf{k} vector N_k , \mathbf{q} vector N_q , and modes N_ν)

Efficient calculation of e-ph vertex:

Coarse grids: reduction of computational size

- Exploit the crystal symmetry to reduce the number of evaluations for e-ph vertex

$$\begin{aligned}g_{mn\nu}(\mathbf{k}, S\mathbf{q}) &= \langle \psi_{m, \mathbf{k}+S\mathbf{q}}(\mathbf{r}) | \partial_{\nu, S\mathbf{q}} V(\mathbf{r}) | \psi_{n, \mathbf{k}}(\mathbf{r}) \rangle \\ &= \langle \psi_{m, \mathbf{k}+S\mathbf{q}}(\mathbf{r}) | \partial_{\nu\mathbf{q}} V(\{S|\mathbf{v}\}^{-1}\mathbf{r}) | \psi_{n, \mathbf{k}}(\mathbf{r}) \rangle \\ &= \langle \psi_{m, \mathbf{k}+S\mathbf{q}}(\{S|\mathbf{v}\}\mathbf{r}) | \partial_{\nu\mathbf{q}} V(\mathbf{r}) | \psi_{n, \mathbf{k}}(\{S|\mathbf{v}\}\mathbf{r}) \rangle\end{aligned}$$

[Phys. Rev. B **76**, 165108 (2007)]

- If the following relations hold throughout all wave functions,

$$\begin{aligned}\psi_{m, S^{-1}\mathbf{k}+\mathbf{q}}(\mathbf{r}) &= \psi_{m, \mathbf{k}+S\mathbf{q}}(\{S|\mathbf{v}\}\mathbf{r}) \\ \psi_{n, S^{-1}\mathbf{k}}(\mathbf{r}) &= \psi_{n, \mathbf{k}}(\{S|\mathbf{v}\}\mathbf{r})\end{aligned}$$

- Can we use the symmetry relation between e-ph vertices?

$$g_{mn\nu}(\mathbf{k}, S\mathbf{q}) = g_{mn\nu}(S^{-1}\mathbf{k}, \mathbf{q})$$

Efficient calculation of e-ph vertex:

Coarse grids: reduction of computational size

- We can't use the equation below to reduce the number of evaluations:

$$g_{mn\nu}(\mathbf{k}, S\mathbf{q}) = g_{mn\nu}(S^{-1}\mathbf{k}, \mathbf{q})$$

- Ex) For the inversion operation in the little group of the wavevector \mathbf{k} , and its odd-parity state n ,

$$\psi_{n\mathbf{k}}(\{S|\mathbf{v}\}\mathbf{r}) = \{S|\mathbf{v}\}^{-1}\psi_{n\mathbf{k}}(\mathbf{r}) = -\psi_{n\mathbf{k}}(\mathbf{r}) \neq \psi_{n,S^{-1}\mathbf{k}}(\mathbf{r})$$

[$S^{-1} = S$ for inversion operation]

- We need to multiply the **additional factors** (the same for states at $\mathbf{k}+\mathbf{q}$):

$$g_{mn\nu}(\mathbf{k}, S\mathbf{q}) = \sum_{m'n'} \langle \psi_{m',S^{-1}\mathbf{k}+\mathbf{q}}(\mathbf{r}) | \psi_{m,\mathbf{k}+S\mathbf{q}}(\{S|\mathbf{v}\}\mathbf{r}) \rangle^* g_{m'n'\nu}(S^{-1}\mathbf{k}, \mathbf{q}) \langle \psi_{n',S^{-1}\mathbf{k}}(\mathbf{r}) | \psi_{n\mathbf{k}}(\{S|\mathbf{v}\}\mathbf{r}) \rangle$$

Efficient calculation of e-ph vertex:

Coarse grids: reduction of computational size

- We can reduce the number of evaluations:
 - For each \mathbf{q}_{irr} , explicitly evaluate the e-ph vertex
 - For the star of \mathbf{q}_{irr} , skip the explicit evaluation by using the symmetry relation of e-ph vertex
- Speedup by a factor of $N_{\mathbf{q}}/N_{\mathbf{q}_{\text{irr}}}$ ($\sim \alpha$)
 - For GaN with 6x6x6 \mathbf{k} and \mathbf{q} points ($N_{\mathbf{q}}/N_{\mathbf{q}_{\text{irr}}}=7.7$) , speedup by 4

Efficient calculation of e-ph vertex:

Fine grids: optimize the computation

- Symmetry broken during Wannierization

$$g_{mn\nu}(\mathbf{k}, \mathbf{q}) = \sum_{pp'} e^{i(\mathbf{k} \cdot \mathbf{R}_p + \mathbf{q} \cdot \mathbf{R}_{p'})} \sum_{m'n'\kappa\alpha} U_{mm'\mathbf{k}+\mathbf{q}} g_{m'n'\kappa\alpha}(\mathbf{R}_p, \mathbf{R}_{p'}) U_{n'n\mathbf{k}}^\dagger u_{\kappa\alpha, \mathbf{q}\nu}.$$

, where Wigner-Seitz vector $\mathbf{R}_{p(p')}$, rotation matrix U , eigenmode u , band index $m(m')$, $n(n')$, mode index ν , and Cartesian directions α

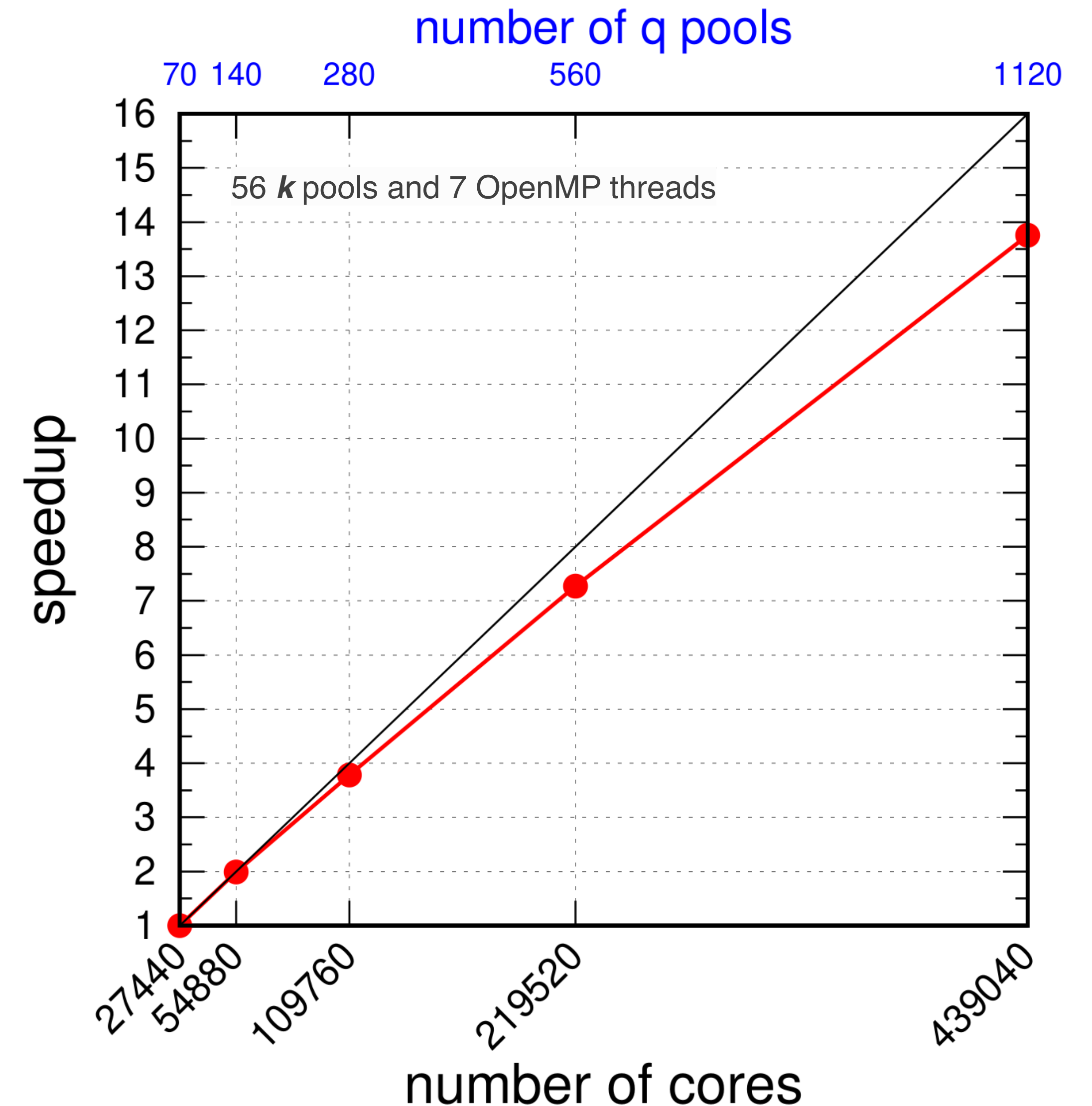
[Phys. Rev. B **76**, 165108 (2007), Rev. Mod. Phys. **89**, 1 (2017)]

- Instead, hybrid parallelization scheme enables highly-scalable evaluations
 - \mathbf{q} point parallelization using MPI
 - \mathbf{k} point parallelization using MPI and OpenMP

Efficient calculation of e-ph vertex:

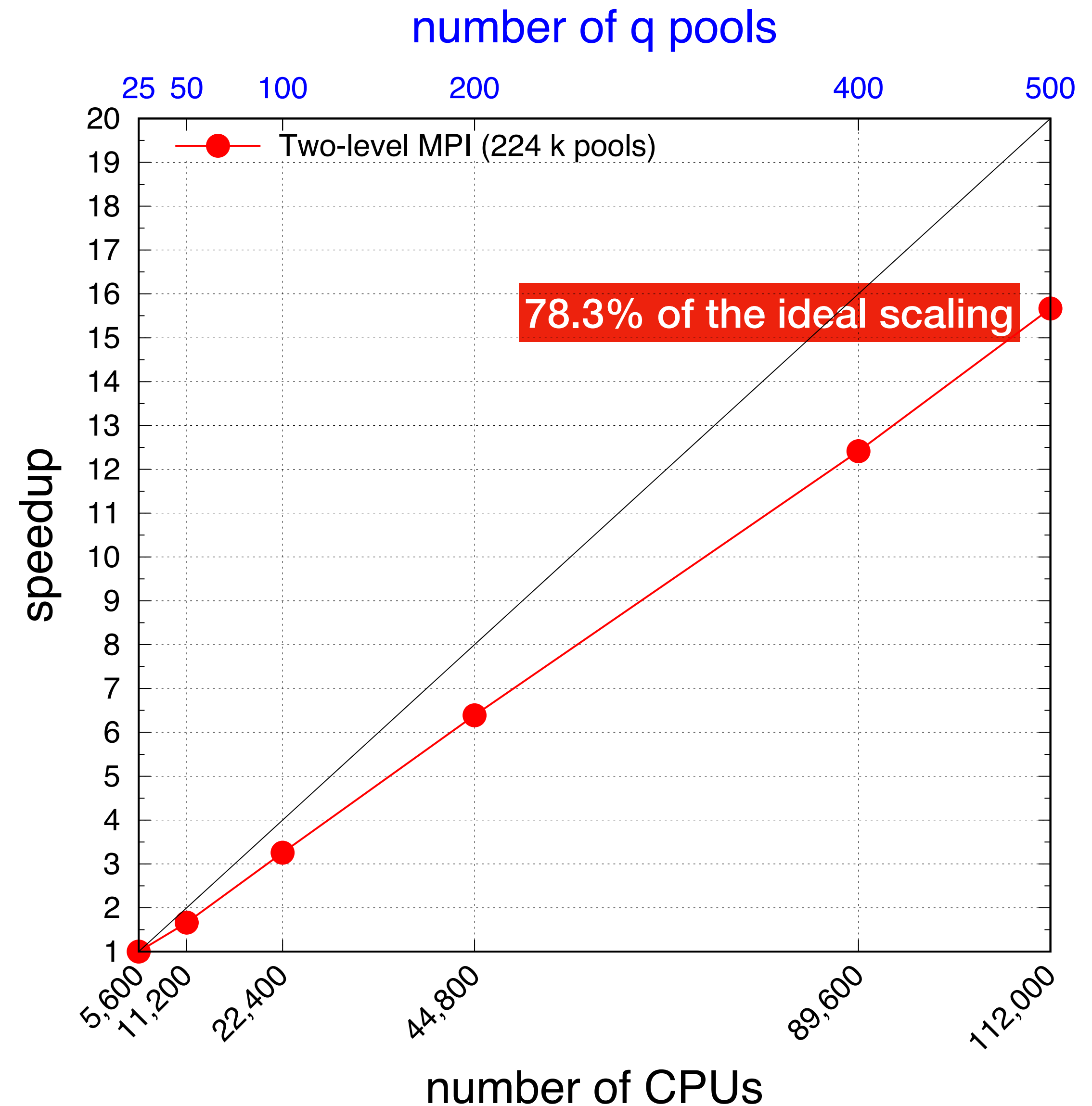
Fine grids: optimize the computation

- Hybrid two-level MPI and OpenMP
- 86% speedup of the ideal one on half-million cores
[done on Frontera during Texascale Days in Dec. 2020]
- Time-to-solution of 21 min. for the interpolation of e-ph vertex on 10^6 full \mathbf{k} and \mathbf{q} points with 3 bands for MgB_2 (8.1×10^{13} elements)



Extension of Hybrid Parallelization Scheme: Superconducting module

- Done on Frontera up to 112,000 cores
- MgB₂
 - 36x36x36 *k* and *q* fine grids (2,413 irreducible *k* points and 46,656 full *q* points)
 - No Fermi window (five bands considered)
 - Maximum number (40) of iterations set for benchmark purpose
- Use of low I/O mode



Other additions

- Symmetric e-ph vertex in USPP and PAW
- OpenMP parallelization for G operations
- Parallel I/O using parallel HDF5
 - Reduce the time for I/O
 - Average time of 15.3 sec for file I/O of 3.3 TB using 5,040 cores
[done on Summit with GPFS file system]
 - Reduce the number of files to minimum
 - Increased portability and flexibility
- Optimized linear tetrahedron method [Phys. Rev. B **89**, 094515 (2014)]

Symmetric e-ph vertex in USPP and PAW

- Only a few correction terms needed to add to ultrasoft pseudopotential (USPP) case [Phys. Rev. B **81**, 075123 (2010)]

$$\sum_{\sigma} \sum_{I,mn} \Delta D_{I,mn}^{1,\sigma,\mu} b_{I,mn}^{\sigma,\lambda}$$

$$\text{where } \Delta D_{I,mn}^{1,\sigma,\mu} = \left(\frac{dD_{I,mn}^{1,\sigma}}{d\mu} - \frac{d\tilde{D}_{I,mn}^{1,\sigma}}{d\mu} \right), \quad \frac{dD_{I,mn}^{1,\sigma}}{d\mu} = \sum_{\sigma_1} \int_{\Omega_I} d^3r \Phi_m^{I,AE}(\mathbf{r}) \Phi_n^{I,AE}(\mathbf{r}) \frac{dV_{\text{eff}}^{I,\sigma}}{d\rho_{\sigma_1}^{1,I}} \frac{d\rho_{\sigma_1}^{1,I}}{d\mu}$$

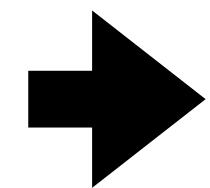
$$, \text{ and } b_{I,mn}^{\sigma,\mu} = \sum_i \langle \tilde{\psi}_{i,\sigma} | \frac{\partial(|\beta_m^I\rangle\langle\beta_n^I|)}{\partial\mu} | \tilde{\psi}_{i,\sigma} \rangle - \sum_i \langle \tilde{\psi}_{i,\sigma} | \beta_m^I \rangle \langle \beta_n^I | \delta^\mu \tilde{\psi}_{i,\sigma} \rangle$$

- Calculation of terms common in both PAW and USPP parallelized over \mathbf{G}

Symmetric e-ph vertex in USPP and PAW

- Use of symmetric expression of e-ph vertex
[Phys. Rev. B **100**, 174304 (2019), Phys. Rev. B **101**, 184302 (2020)]

$$g_{mn\nu}(\mathbf{k}, \mathbf{q}) = \underbrace{\langle \tilde{\Psi}_{m, \mathbf{k}+\mathbf{q}} | \partial_{\nu\mathbf{q}} \tilde{H} - \epsilon_{n\mathbf{k}} \partial_{\nu\mathbf{q}} \tilde{S} | \tilde{\Psi}_{n\mathbf{k}} \rangle}_{g_{mn\nu}^{\text{QE}}(\mathbf{k}, \mathbf{q})} + (\epsilon_{n\mathbf{k}} - \epsilon_{m, \mathbf{k}+\mathbf{q}}) \langle \tilde{\Psi}_{m, \mathbf{k}+\mathbf{q}} | \hat{T}^\dagger \partial_{\nu\mathbf{q}} \hat{T} | \tilde{\Psi}_{n\mathbf{k}} \rangle$$

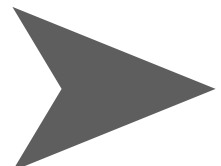


$$|g_{mn\nu}(\mathbf{k}, \mathbf{q})| = |g_{nm\nu}^*(\mathbf{k} + \mathbf{q}, -\mathbf{q})|$$

- For use with USPP as well, ignore the negligible term with derivatives of on-site partial waves

New Parallel Structure

$$g(\mathbf{k}_c, \mathbf{q}_c) \rightarrow g(\mathbf{R}_e, \mathbf{R}_p)$$

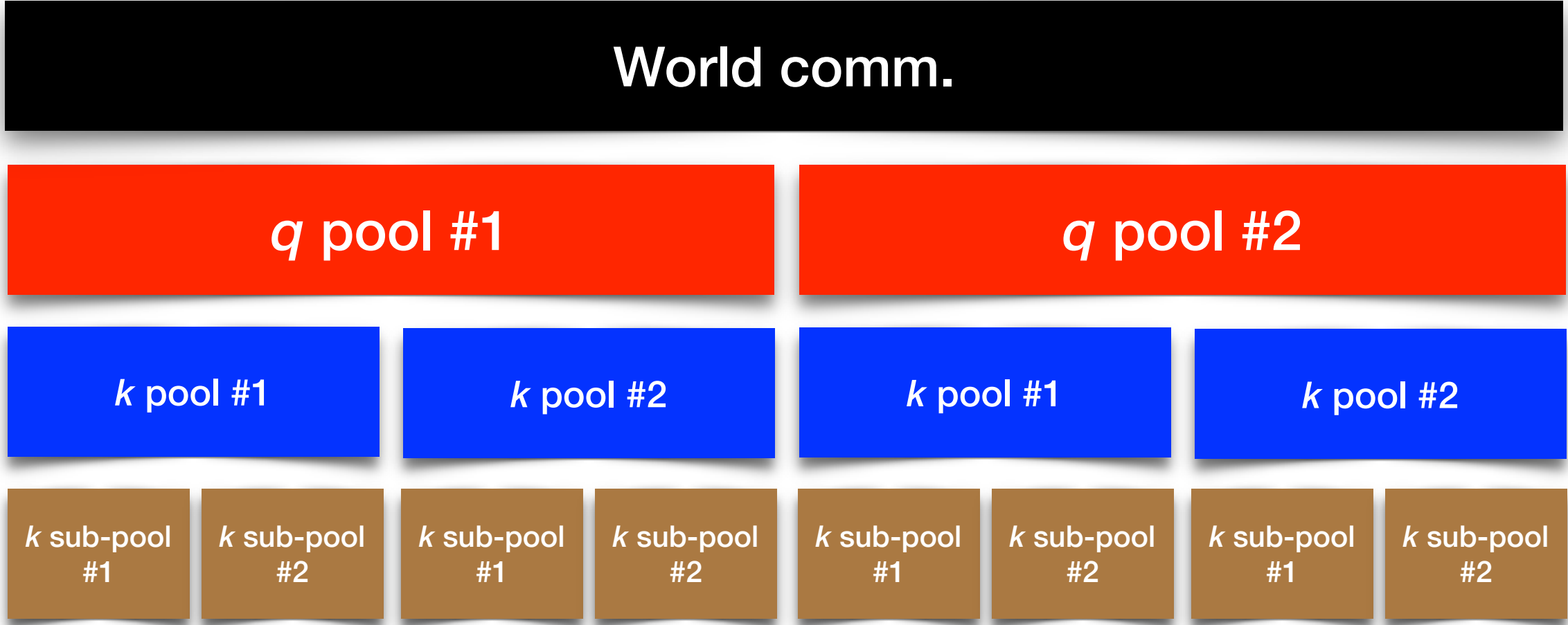
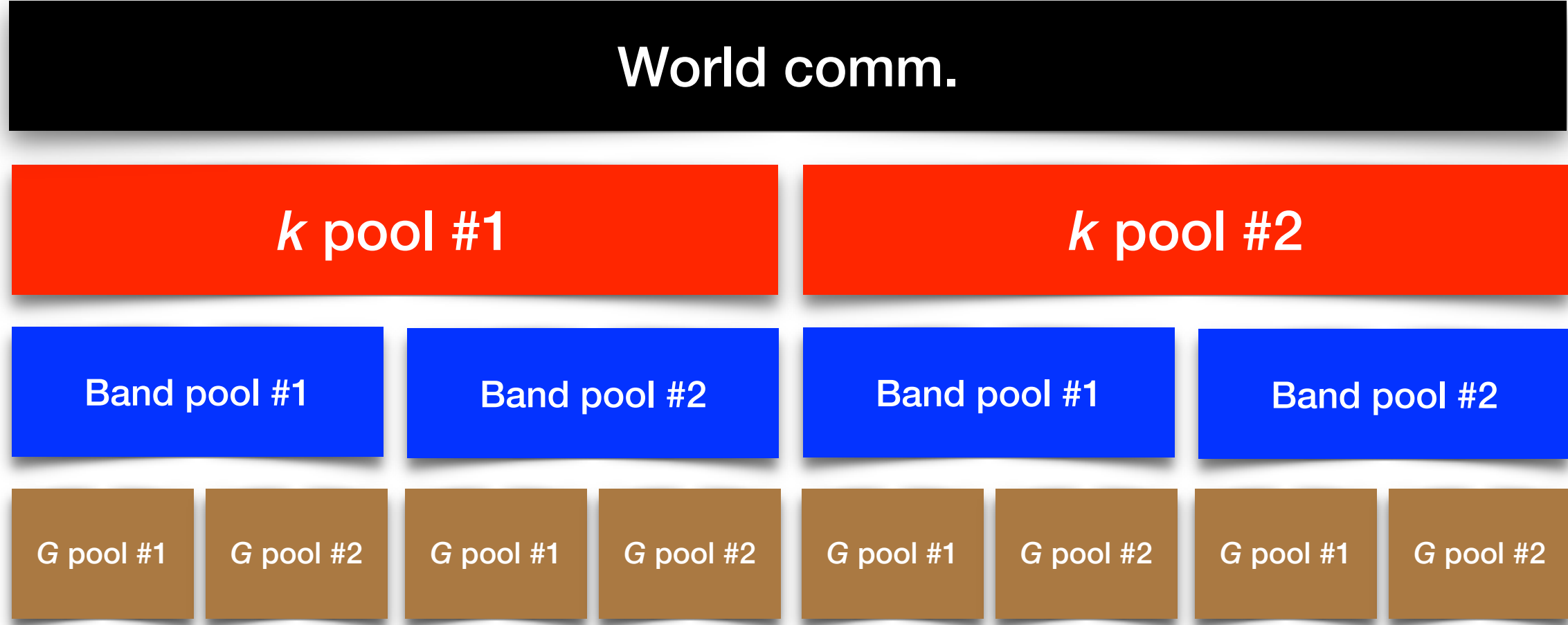


$$g(\mathbf{k}_f, \mathbf{q}_f)$$



$$\sum''_{n\mathbf{k}_f} [g(\mathbf{k}_f, \mathbf{q}_f)],$$

$$\prod''_{\nu\mathbf{q}_f} [g(\mathbf{k}_f, \mathbf{q}_f)], \dots$$



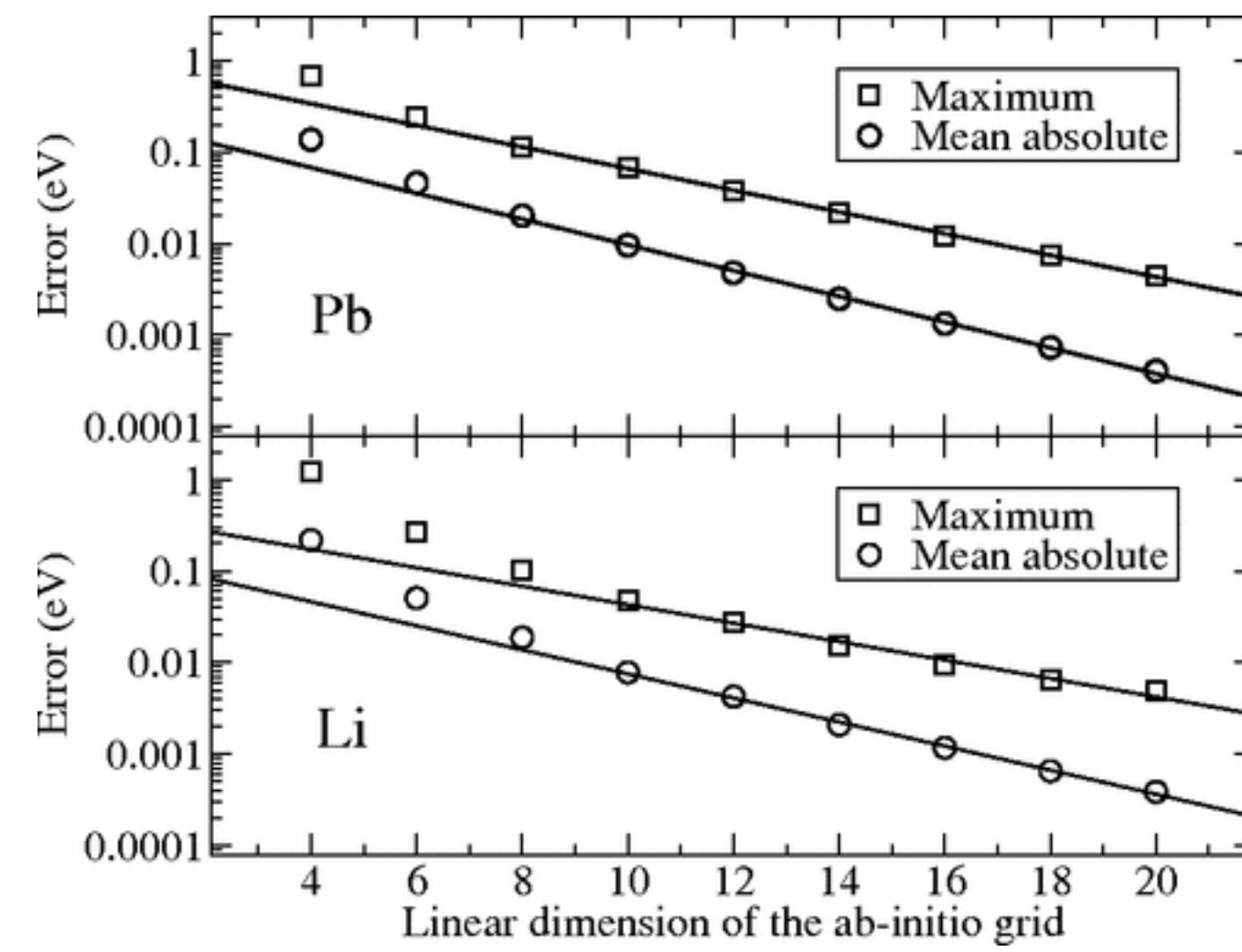
Short-term plans

- Easy construction of (symmetric) localized basis
- Non-collinear magnetism

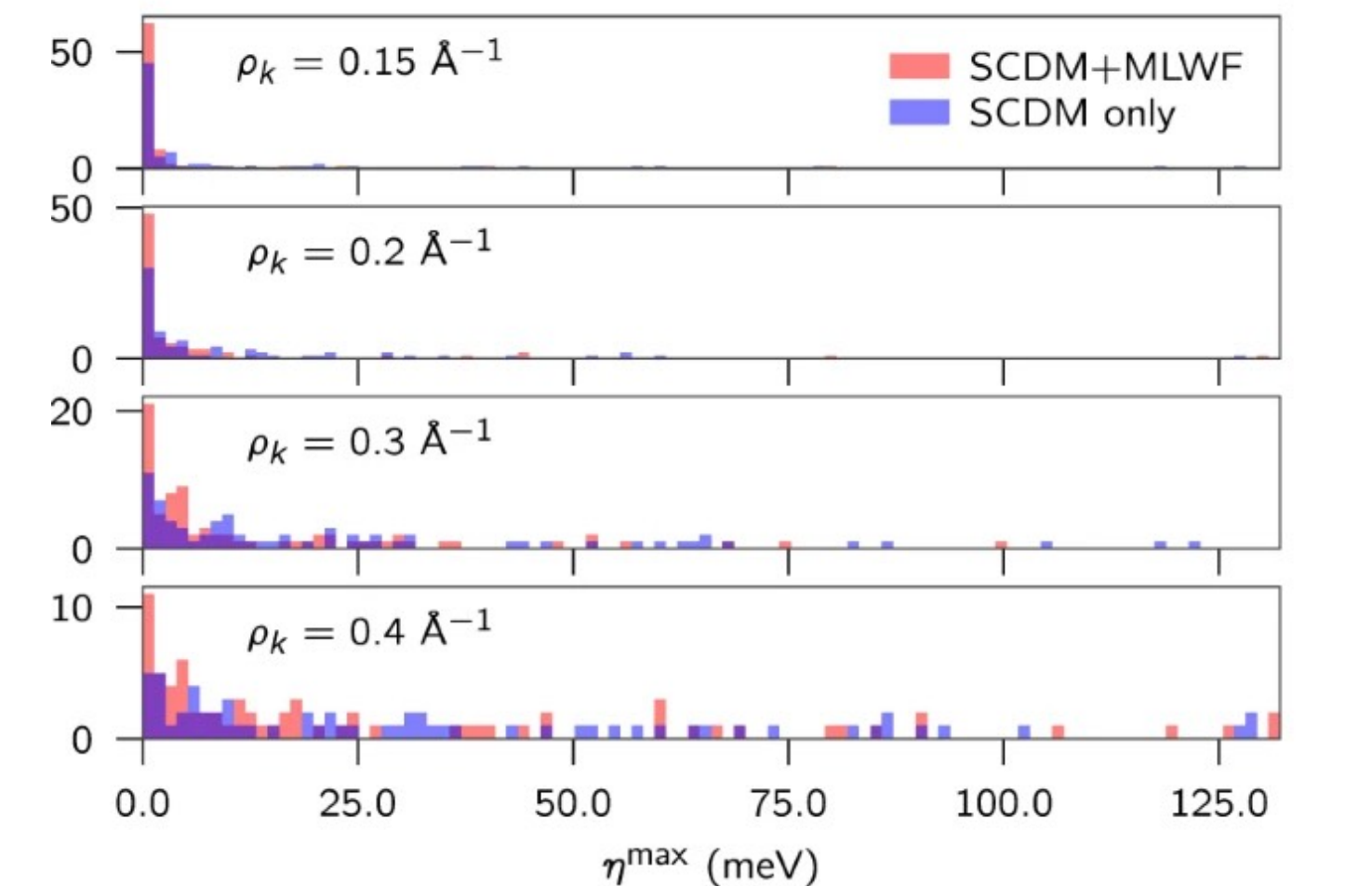
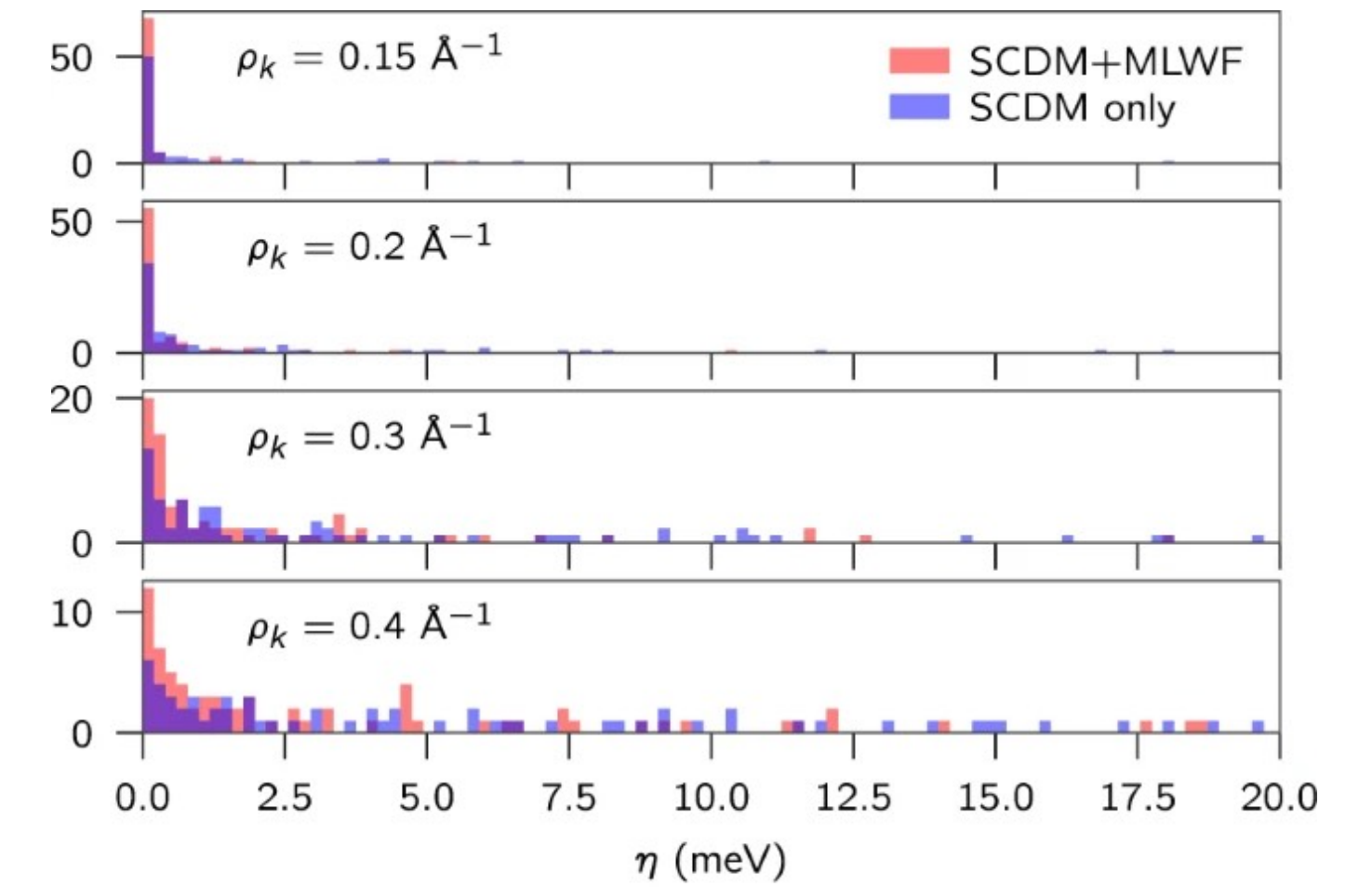
Short-term plans

Easy construction of (symmetric) localized basis

- Importance of high grid density



[Phys. Rev. B 75, 195121 (2007)]



[npj Comput Mater 6, 66 (2020)]

Short-term plans

Easy construction of (symmetric) localized basis

- Projectability [Phys. Rev. B **88**, 165127 (2013)]
- Optimal choice of SCDM-k parameters [npj Comput Mater **6**, 66 (2020)]
- Energy window optimization [Phys. Rev. Materials **2**, 103805 (2018)]

Mid-term plans

- Port EPW to the heterogeneous architecture (GPU)
- Interface EPW to other DFT codes

Mid-term plans

Porting EPW to GPU

- Importance of OpenMP
- Choice of programming models

An Overview of Some DOE Systems

	Delivery	CPU + Accelerator Vendor
Summit	2018	IBM + NVIDIA
Sierra	2018	IBM + NVIDIA
<i>Perlmutter</i>	2020	AMD + NVIDIA
Aurora	2021	Intel + Intel
Frontier	2021	AMD + AMD
El Capitan	2022	AMD + AMD

<https://alcf.anl.gov/support-center/aurora/porting-aurora>

References

- F. Giustino et al., Phys. Rev. B 76, 165108 (2007) [\[link\]](#)
- C. Verdi et al., Phys. Rev. Lett. 115, 176401 (2015) [\[link\]](#)
- S. Ponc'e et al., Comput. Phys. Commun. 209, 116 (2016) [\[link\]](#)
- F. Giustino, Rev. Mod. Phys. 89, 1 (2017) [\[link\]](#)

More Info



- <https://epw-code.org>



- <https://forum.epw-code.org>



- <https://gitlab.com/QEF/q-e>

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