GW, GW-BSE, and beyond: Quasiparticle excitations, optical responses, and field-driven time-dependent phenomena

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> **Supported by: Department of Energy National Science Foundation**

> > **School on Electron-Phonon Physics, Many-Body Perturbation Theory, and Computational Workflows** June 13, 2024

- Excited-state phenomena (transport, optical, energy transfer, charge/spin dynamics...) in a material often give rise to its *defining attributes* and determine its usefulness.
- *Ab initio* calculations of excited states \implies different and higher-level treatment of many-electron interaction effects
- Existing *ab initio* methods and community codes for solids: predict & understand 1-particle or 2-particle excitation phenomena in increasingly complex materials. *(BerkeleyGW, WEST, Yambo, VASP, ABINIT, FHI-aims, Exciting, stochasticGW, nanoGW,* FlapwMBPT, *QMCPACK, EPW,...)*
- Key phenomena correlated 3- or 4-particle excitations, field-driven timedependent processes, non-linear spectroscopies, excited-state dynamics, etc. – remain to be addressed efficiently by ab initio methods & public domain software, including relevant & accurate many-electron interactions.

Study of Material Properties – A N-Particle Quantum Problem

$$
H_T = \sum_{i} \frac{p_i^2}{2m} + \sum_{n} \frac{p_n^2}{2M_n} + \frac{1}{2} \sum_{ij} \frac{e^2}{|\mathbf{r}_i - \mathbf{r}_j|} + \frac{1}{2} \sum_{nn'} \frac{Z_n Z_{n'} e^2}{|\mathbf{R}_n - \mathbf{R}_{n'}|} + \sum_{n,i} V_n(\mathbf{r}_i - \mathbf{R}_n) + H_R,
$$

\n
$$
\Psi = \Psi(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3... \mathbf{r}_N, t)
$$

\nFor large N (~10²²/cc),
\n• impossible to solve exactly
\n• often not desirable to get full solutions
\nN+1 particle

problem

E.g., single-particle behaviors:
 $G(\mathbf{r}, \mathbf{r}', \tau) = -i \langle 0 | T \{ \psi(\mathbf{r}, \tau) \psi^{\dagger}(\mathbf{r}', 0) \} | 0 \rangle$

Optical properties – N+2 particle problem

Interacting single-particle Green's function

Electrons in Crystals

- **Interacting many-electron problem**
	- Need many-body techniques for electronic, transport and optical properties
	- Moderately correlated systems may be solved numerically from *first principles* using many-body perturbation theory (e.g., GW and GW-BSE approaches, and beyond)

$$
\frac{1}{\sqrt{1-\frac{1}{2}}\cdot\frac{1}{2}} = iGW
$$

BSE kernel

1-particle excitations (N+1 particle problem) Optical excitations (N+2 particle problem)

- Introduction -- many-body interactions and excitedstate phenomena in materials
- Quasiparticle excitations, optical responses, and correlated multiparticle excitations
- Time-dependent and nonlinear phenomena
- Photophysics of moiré 2D heterostructures
- Correlation enhanced electron-phonon interactions

Ab initio **Studies of Quasiparticle Excitations in Tunneling and ARPES**

1-particle excitations (N+1 particle problem)

$$
E(\mathbf{k}) = E^{0}(\mathbf{k}) + \blacktriangleleft (\mathbf{k}, E)
$$

- renormalizes energy dispersion
- gives rise to finite lifetime
- satellite structures

$$
A(\mathbf{k}, \omega) = \frac{2}{\pi} \frac{-\mathrm{Im} \Sigma(\mathbf{k}, \omega)}{\left[\omega - \varepsilon_{\mathbf{k}} - \mathrm{Re} \Sigma(\mathbf{k}, \omega)\right]^2 + \left[\mathrm{Im} \Sigma(\mathbf{k}, \omega)\right]^2}
$$

e-e⁺ e-ph +…

Quasiparticle (electron or hole) excitations in solids: the GW approach

 $E(\mathbf{k}) = E^{0}(\mathbf{k}) + \langle \mathbf{k}, \mathbf{k} \rangle$ Interacting 1-particle Green's Functions:

$$
W(\vec{r}',\vec{r}',\omega) = \int v(\vec{r},\vec{r}'')\epsilon^{-1}(\vec{r}'',\vec{r}',\omega)d^3r''
$$

Screened Coulomb interaction

Hubbard (1957) Phillips (1961)

Interacting 2-particle Green's function via the Bethe-Salpter equation (GW-BSE): linear optical properties including excitonic effects

GW approximation:

 Electron gas - Hedin, PR (1965) Real materials - Hybertsen and Louie, PRL (1985)

Quasiparticle Band Structure Calculations

• Density-functional theory:

$$
\left\{-\nabla^2 + V_{\text{ext}} + V_{\text{Coul}} + V_{\text{xc}}\right\}\psi_{\text{n}\mathbf{k}}^{\text{DFT}} = \varepsilon_{\text{n}\mathbf{k}}^{\text{DFT}}\psi_{\text{n}\mathbf{k}}^{\text{DFT}}
$$

Hohenberg, Kohn, and Sham 1965

• Green-function approach

$$
\left\{-\nabla^2+V_{\text{ext}}+V_{\text{Coul}}+\Sigma(\varepsilon^{\text{QP}}_{\text{n}\textbf{k}})\right\}\psi^{\text{QP}}_{\text{n}\textbf{k}}=\varepsilon^{\text{QP}}_{\text{n}\textbf{k}}\psi^{\text{QP}}_{\text{n}\textbf{k}}
$$

 $\Sigma = iG_1W$ GW approximation for the self energy

 G_1 one-particle Green function $W = e^{-1}v$ screened Coulomb interaction

Hedin 1965, Hybertsen and Louie 1985

- $\Sigma(\mathbf{r}, \mathbf{r}', \omega)$ nonlocal, energy-dependent nonHermitian operator
- Large complex eigenvalue problem

The GW approximation for Σ

$$
\Sigma(\vec{r}, \vec{r}'; E) = \frac{i}{2\pi} \int W(\vec{r}, \vec{r}', \omega) G(\vec{r}, \vec{r}', E + \omega) e^{i\delta\omega} d\omega
$$

with

$$
W(\vec{r}, \vec{r}', \omega) = \int v(\vec{r}, \vec{r}'') \varepsilon^{-1} (\vec{r}'', \vec{r}', \omega) d^3 r''
$$

$$
G(\vec{r}, \vec{r}', \omega) = \sum_{n \vec{k}} \frac{\Psi_{n\vec{k}}(\vec{r}) \Psi_{n\vec{k}}(\vec{r}')}{\omega - E_{n\vec{k}}(\vec{r}, \omega)}
$$

 \mathbf{m} $-nk$ $n\mathsf{k}$

 $H = H_0 + (H - H_0)$

…

 H_o : LDA/GGA LDA+U exact exchange hybrid functionals iterative scheme

Require:

(1) Full dielectric matrix (local fields) $\epsilon^{-1}(\vec{r}', \vec{r}', \omega)$ or $\epsilon^{-1}_{\vec{G}\vec{G}'}$ (\vec{q}', ω) (2) Good starting $\psi_{n\vec{k}}$ and $E_{n\vec{k}}$ to construct the Green's function G.

Hybertsen and Louie (PRL 1985)

Calculation of Optical Absorption Spectrum

• Coupled electron-hole excitations:

$$
\left| S \right> = \tfrac{\text{elec hole}}{v} \tfrac{S}{c} A_{vc}^{S} \, \hat{a}^{\dagger}_v \, \hat{b}^{\dagger}_c \left| 0 \right>
$$

 $|0\rangle$ ground state of many-electron system \hat{a}_{v}^{\dagger} , \hat{b}_{c}^{\dagger} creates quasi-hole, -electron A_{vc}^{S} coupling coefficients $v = n\mathbf{k}$ (occupied) $c = n'$ **k** (unoccupied)

• The Bethe-Salpeter Equation for the two-particle Green's function G_2 yields:

$$
(\varepsilon_{c}^{\textrm{QP}}-\varepsilon_{v}^{\textrm{QP}})A_{vc}^{S}+\underset{v'c'}{\Sigma}\langle vc|K^{eh}|v'c'\rangle A_{v'c'}^{S}=\Omega^{S}A_{vc}^{S}
$$

M. Rohfling and S. G. Louie, PRL (1998)

 ε_c^{QP} , ε_v^{QP} single-quasiparticle energies K^{eh} electron-hole interaction

 $\implies \Omega^S$ excitation energies

- $\bullet \Longrightarrow$ Optical absorption spectrum: $\epsilon_2(\omega)$
	- $\ln \text{m}$ • K - large, nonHermitian matrix with very fine **k** sampling
	- K can have dimensions as large as $\sim 10^6$

1) Hybertsen and Louie, PRL (1985); Louie, Topics in Comput. Mat. Sci. (1997)

2) Rohlfing and Louie, PRL (1998); Deslippe, Samsonidze, Strubbe, Jain, Cohen, and Louie, Comput. Phys. Commun. (2012)

Large-scale ab initio GW Calculations over the Years

Large-scale ab initio GW Calculations over the Years

[1] Hybertsen and Louie, PRL 55, 1418 (1985).

[2] Del Ben, Yang, Li, Jornada, Louie, Deslippe, in 2020 SC20: International Conference for High Performance Computing, Networking, Storage and Analysis (SC), Atlanta, GA, US, 2020 pp. 36-46

Quantum level alignment at interface – importance of nonlocal screening/correlations

Energy level diagram

Physical effects

- Interfacial charge transfer (dipoles)
- Quantum mechanical (electronic) coupling
- Electron correlation (**self energy**) effects

Benzene orbital energies on graphite

•**DFT-LDA gap: ~2 eV error**

Neaton, Hybertsen, Louie, PRL **97**, 216405 (2006)

- **Relative to gas phase: Gap narrows by ~3.2 eV**
- **Kohn-Sham energies do not capture the change**
- *In general***, change in self energy: large (~ 2 eV) and environmental dependent**

HOMO - LUMO Quasiparticle Gap of Benzene on Surfaces

Strong & *different* renormalization of molecular levels with different contacts are *absent* in the Kohn-Sham orbital energies -- LDA, GGA or hybrids!!

Electron-Electron & Electron-Hole Interactions in Quasi 2D Systems

Substrate/environmental screening are important!

Optical Spectrum of Monolayer MoS2: GW-BSE Theory vs Experiment

Substrate Screening: Monolayer MoSe² on bilayer graphene

Induced charge due to substrate

- Screening is nonlocal, frequency dependent
- Incorporate screening of *substrate* by via substrate dielectric matrix $\epsilon_{\text{Gz.Gz'}}(\mathbf{q},\omega)$

Jornada, Qiu and Louie (2014)

Substrate Screening: Monolayer MoSe² on bilayer graphene

due to substrate

For some systems, substrate/encapturation can

- renormalize the band gap by **>50%**
- change the exciton binding energies by **factor of 2 or more**
- tune the energy (\sim 1eV) and dispersion of long-live plasmons in monolayer metallic TMDs by an order of magnitude

E.g., Ugeda, Crommie, Wang, Louie, et al, Nat. Mat. (2014); Li, et al, Nat. Nanotech., $(2017), ...$

da Jornada, Xian, Rubio, Louie (2018)

Current BerkeleyGW package - *www.berkeleygw.org*

General excited-state phenomena code that supports:

- GW & GW-BSE: 3D, 2D, 1D and molecular systems
- Insulating, metallic and semi-metallic systems
- Interface with a large set of DFT and other mean-field codes: PARATEC, Quantum Espresso, PARSEC, SIESTA, Octopus, ABINIT, RMGDFT, EPW, …
- Massively parallel with MPI, OpenMP and SIMD parallelism
- Linear scales to *500,000 CPUs* @ NERSC's Cori2, & to **27,648** GPUs @ OLCF's Summit
- Efficient accurate solution to the BSE via **k**-point interpolation (up to *1,000,000 k-points – important for excitons!*)
- Example of recent large-system studies
	- QP states & optical excitations of defects in Si and SiC with **>2,740-atoms** (or **10,900-electrons**) supercell

BerkeleyGW scales linearly up to **512,000** CPU cores!

Non-equilibrium and beyond 1- & 2-particle excitations

New Phenomena \longrightarrow New *ab initio* formalisms

Beyond ab initio GW and GW-BSE

…

Correlated Higher-number-particle Excitations:

- Bi-excitons, trions, exciton-exciton interactions, decay of excited state, …
- Interacting 3- and 4-particle Green's functions,

Time-dependent & High-field Phenomena

- Pump-probe experiments, nonlinear optics, fielddriven transformations, …
- Explicit time propagation of Green's function (time-dependent GW, non-equilibrium Green's function formalism, …)

Correlated Multi-particle Excitations in 1D & 2D

- Multi-particle excitations prominent in atomically thin $1D \& 2D$ systems
- Example: biexcitons on monolayer $WSe₂$
- Trions & biexcitons interacting 3- and 4-particle Green's functions

Other fundamental and practical interests:

- Exciton condensates/insulators
- Singlet fission in organic crystals

Z. Ye, T. Heinz, et al., Nat. Commun. (2018) (Monolayer WSe2 encap. w/ hBN)

Correlated Multi-particle Excitations

• Quasiparticle excitations -- GW approach (1 diagram)

• Optical (electron-hole) excitations – GW-BSE approach (2 diagrams)


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\Sigma = iGW
```
- Optical (electron-hole) excitations - GW-BSE approach

• Bi-excitons (4-particles excitations) – present approach (36 diagrams!)

+ 35 other ones

da Jornada, Cepellotti & Louie (2020)

Trions and Biexcitons in Monolayer WSe2: Theory vs. Experiment

 $+\omega_1$

 $+\omega$ ₂

important e-h

exchange effects

K

K'

 Γ

 $X^{0}X^{i}({\bf Q} = K')$ $X^0 X^{D'} (Q = 0)$ $X^0 X^{D'} (Q = 0)$ $X^0X^i(Q = K')$ Ignored previously owing to neglect of

• Fully *ab initio* many-body (N+3 & N+4 particles) calculations

- Very rich energy and spin/valley level structures (high temp phenomena)
- Elucidate experimental observations + new predictions

[1] He, Shan, et al, PRL 113, 026803 (2014). [2] Ye, Heinz, et al, Nat. Commun. 9, 3718 (2018). [3] Li,, Shi, et al, Nat. Commun. 9, 3719 (2018). [4] Yong, Wang, et al, Nat. Phys. 14, 1092 (2018).

Time-dependent phenomena and nonlinear optical response: TD-GW

- Time propagation \Rightarrow linear/nonlinear responses and time-resolved phenomena with self-energy and exciton effects
- Use an *ab initio* time-dependent adiabatic GW (TD-aGW) approach with real-time propagation of the density matrix²

¹Kadanoff and Baym, Quantum Statistical Mechanics; Keldysh, Zh. Eksp. Teor. Fiz. 47, 1515 (1964) ²Chan, Qiu, da Jornada, and Louie, Proc. Natl. Acad. Sci. U.S.A. **118**, e1906938118 (2021)

TD-aGW Approximation

- TD-aGW approach
	- Accurate in weak & moderate field regime
	- Linear response is equivalent to $BSE¹$
	- Time propagation \Rightarrow nonlinear responses and time-resolved phenomena with exciton effects
- **Can perform propagation with a single time variable²!**

²C. Attaccalite, M. Gruning, A. Marini, PRB 84, 245110 (2011) ¹G. Strinati, Riv. Nuovo Cimento 11, 1 (1988): D. Rocca, D. Lu, and G. Galli, J. Chem. Phys. 133, 164109 (2010)

Pump field-driven phenomena in tr-ARPES

J. K. Freericks, H. R. Krishnamurthy, and Th. Pruschke, PRL 102, 136401 (2009) Photoemission intensity $P(\omega,t)$ $P(\omega, t) \propto \iint$ $t_{\scriptscriptstyle 0}$ t $dt_{_1}dt_{_2}e^{i\,\omega(t_{_2}-t_{_1})}G(t_{_1,}t_{_2})\,.$ $\breve{\mathbf{r}}$

Optical-Field Pumped Spectral Function: Monolayer GeS

Without e-h interaction Chan, Qiu, da Jornada & Louie, *PNAS* 120, e2301957120 (2023) With e-h interaction

Optical-Field Pumped Spectral Function: Monolayer MoS₂

Shift Currents

- Photo-induced DC current in noncentro-symmetric crystals without p-n junction (bulk photovoltaic effect)
- Second-order optical response (related to band topology)
- Excitonic effects in 2D semiconductors can enhance shift currents by orders of magnitude¹

1Chan, Qiu, da Jornada, and Louie, PNAS 118, e1906938118 (2021)

$$
(\omega=0)=2\sigma_{\text{shift}}\mathbf{E}(\Omega)\mathbf{E}^*(\Omega)
$$

Shift of intracell coordinates from optical transitions

W. Kraut and R. von Baltz, PRB 19, 1548 (1979); B. S. I. Sturman and V.M. Fridkin, Photovoltaic and photorefractive effects in noncentrosymmetric materials (1992)

$$
R_{\mathbf{k}\,nm}^{a,b} = \left(\partial_{k_a}\phi_{\mathbf{k}\,nm}^b + A_{\mathbf{k}\,nn}^a - A_{\mathbf{k}\,mm}^a\right)
$$

TD-aGW Approximation: Shift Currents in Monolayer GeS

- **Excitons enhance shift currents; effect is** *huge orders of magnitude higher* **(also other NLO responses such as SHG)**
- **DC charge transport with CW in-gap excitations!**

Chan, Qiu, da Jornada and Louie, Proc. Natl. Acad. Sci. U.S.A. **118**, e1906938118 (2021)

Second Harmonic Generation in Monolayer GeS

Absolute value of $\chi^{(2)$ yyy

Chan, Qiu, da Jornada and Louie, PNAS **118**, e1906938118 (2021)

Moire Superlattices of Twisted Bilayer 2D Materials

Many exciting new physical phenomena

Twisted bilayer TMDs

Moiré pattern Credit:

Kyle Seyler

• Flat bands

…

- Unconventional superconductivity
- Correlated insulating states Mott insulators, Wigner electron crystals…
- Novel moiré excitons in TMD bilayers

Excitons in twisted bilayer transition metal dichalcogenides (TMD)

Intralayer moiré excitons in WSe2/WS²

Type II band alignment

C. Jin, E. C. Regan, A. Yan,…, S. Tongay, A. Zettl and F. Wang, Nature, 567, 76–80 (2019) Also, Y. Tang, L. Li, T. Li, …, A. H. MacDonald, J. Shan and K.F. Mak, Nature 579, 253 (2020)

Naik, Regan, Zhang, … da Jornada, Wang, and Louie, Nature **609**, 52 (2022)

Current vortex crystals from exciton III with different linear polarizations

- \triangleright Two current vortices in each moiré supercell (same magnitude & opposite chirality)
- \triangleright Different light polarization can control the shift current vortex

Hu, Naik, Chan, Ruan and Louie, PNAS 120, e2314775120 (2023)

Shift current vortices: Linear polarization direction dependence

Light polarization direction can tune the shift current vortices continuously (**locations**, **shapes**, **distributions, circulations…**)

Circularly-polarized light illumination \Rightarrow **Ferrimagnetic vortex crystals**

All-optical control of shift current vortex crystals

- Orbital **J**(**r**) of a 2D material is a vector field on a 2 torus $\frac{1}{\sqrt{1}}$ net winding numbers of current vortices in a unit cell should be zero (Poincaré–Hopf theorem): vortices of opposite circulation and ferrimagnetism
- Second-order response: $B \propto J \propto E^2 \propto$ light intensity

Expt intensity: $I \sim$

~ hundreds of ~ hundreds of (Earth's magnetic field: \sim)

Incident light

(frequency, intensity, polarization)

(magnitude, location, shape, chirality) **Generation/manipulation of current vortices & magnetism**

Hu, Naik, Chan, Ruan and Louie, PNAS 120, e2314775120 (2023)

Ab initio **Electron-phonon coupling**

Electron-phonon (*e***-ph) coupling plays a key role in many phenomena**

- Electrical and thermal transport
- Bardeen-Cooper-Schrieffer theory for superconductivity
- Optical spectra (phonon-assisted)
- Charge-density wave
- Hot carrier dynamics
- …

Electron-phonon Coupling in GW perturbation theory (GWPT)

• Standard DFT e-ph matrix elements fail in some materials.

1.4

 1.2

 1.0

 0.8

 0.6

 0.4

 0.2

 0.0

 \prec

 \bullet GW level e-ph matrix elemer $\bm{g}^{GW}_{mn\nu}(\mathbf{k},\mathbf{q})\!=\!g^{DFT}_{mn\nu}(\mathbf{k},\mathbf{q})\!-\!\langle \bm{\psi}_{m\mathbf{k}+\mathbf{q}}|\partial_{\mathbf{q}\,\nu}V_{\text{xc}}|\bm{\psi}_{n\mathbf{k}}\rangle\!+\!\langle \bm{\psi}_{m\mathbf{k}+\mathbf{q}}|\partial_{\mathbf{q}\,\nu}\Sigma|\bm{\psi}_{n\mathbf{k}}\rangle$ LDA/GGA GW many-electron (GWPT)

single-electron XC

self-energy

• Correlation renormalization of g can be as large as a factor of 2!

Few take-home messages

- Many-body interactions are centrally important in materials properties and phenomena.
- Treatment of interaction effects at the appropriate level is essential, especially for excited states and lower dimensional systems.
- Many-body perturbation theory with screened Coulomb interaction to *n*-particle Green's functions has proven to be a powerful & versatile approach for *ab initio* understanding and prediction of a variety of spectroscopic properties and field-driven, time-dependent phenomena in real materials.

Thank you!