

2023 Virtual School on Many-Body Calculations  
using **EPW** and **BerkeleyGW**

5-9 June 2023



U.S. DEPARTMENT OF  
**ENERGY**

**TACC**  
TEXAS ADVANCED COMPUTING CENTER

Lecture Wed.1

# Theory of polarons

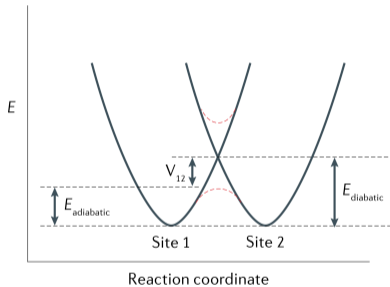
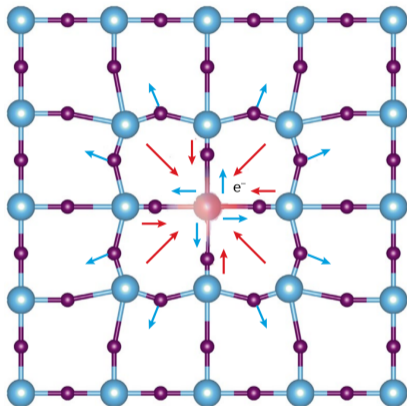
Feliciano Giustino

Oden Institute & Department of Physics

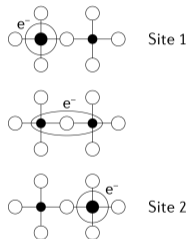
The University of Texas at Austin

- Introduction to the polaron concept
- Photoemission spectra
- DFT calculations of polarons
- Landau-Pekar theory
- *Ab initio* polaron equations
- Examples of polarons
- Many-body theory of polarons

# Intuitive notion of polaron



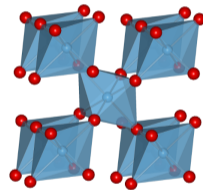
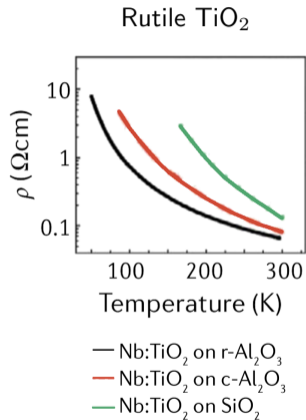
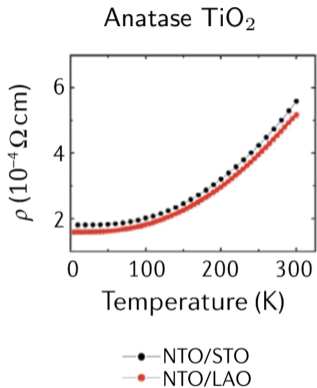
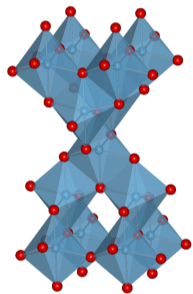
Structural distortions



Figures from Franchini et al, Nat. Rev. Mater. 2021, 10.1038/s41578-021-00289-w

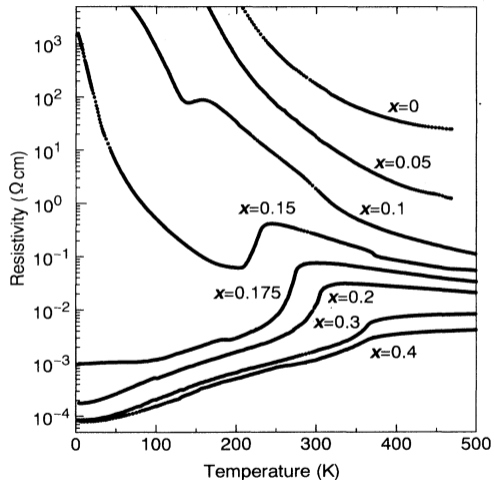


# Transport signatures of polarons



Hall mobility data from Zhang et al, J. Appl. Phys. 102, 013701 (2007)

# Transport signatures of polarons



$\text{La}_{1-x}\text{Sr}_x\text{MnO}_3$

Figure from Urushibara, Moritomo, Arima, Asamitsu, Kido, Tokura, Phys. Rev. B 51, 14103 (1995)

# Polarons in photoelectron spectroscopy

## Angle-resolved photoelectron spectroscopy (ARPES)

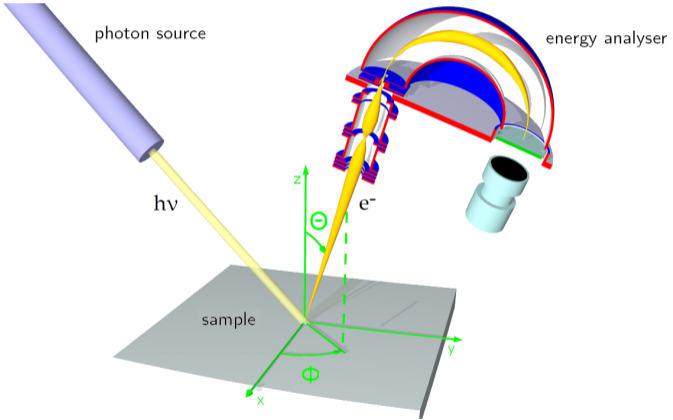


Figure from commons.wikimedia.org/wiki/File:ARPESgeneral.png

# Phonon satellites in anatase TiO<sub>2</sub>

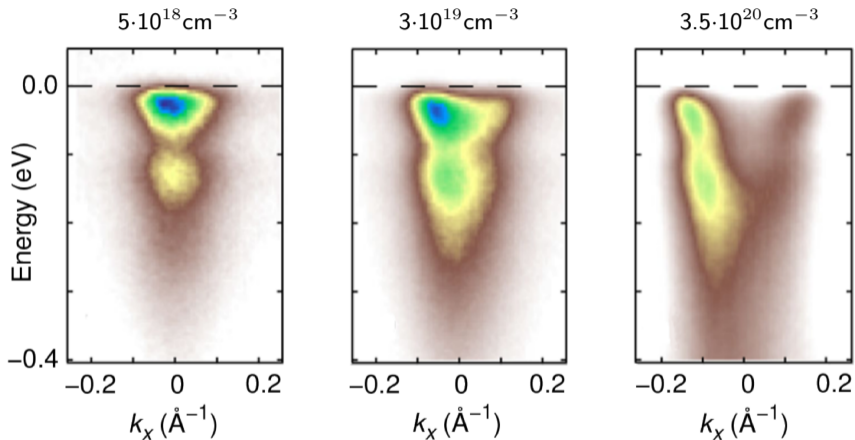


Figure from Moser et al, Phys. Rev. Lett. 110, 196403 (2013)

# Phonon satellites in EuO

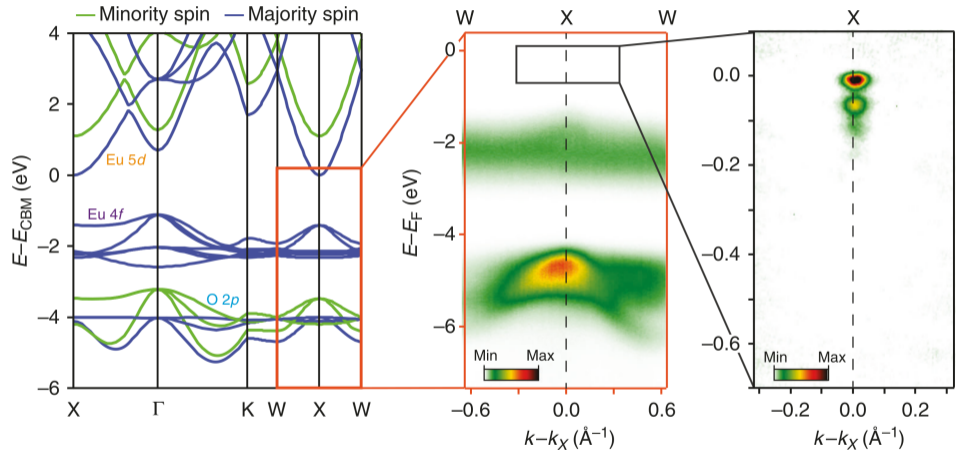
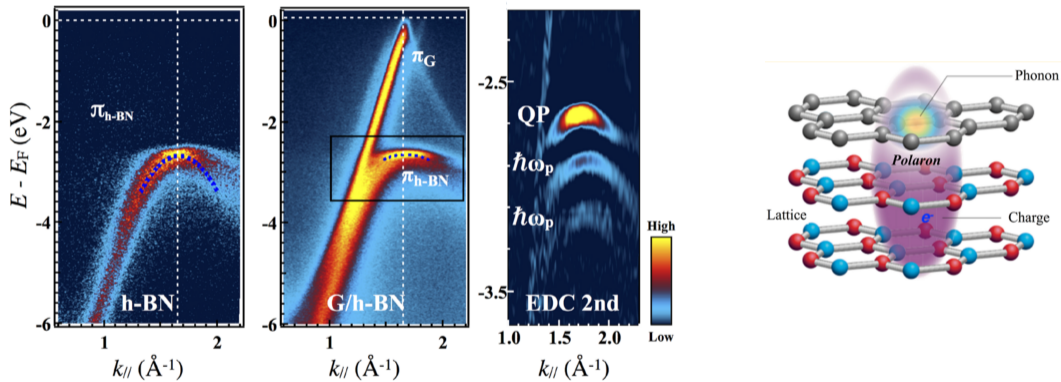


Figure from Riley et al, Nat. Commun. 9, 2305 (2018)

# Phonon satellites in 2D h-BN



Figures from Chen et al, Nano Lett. 18, 1082 (2018)

# Polaron satellites (aka phonon sidebands)

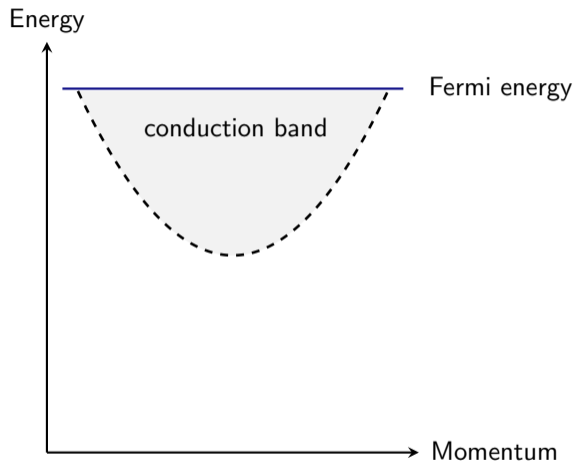


Figure adapted from Kandolf et al, Phys. Rev. B 105, 085148 (2022)

# Polaron satellites (aka phonon sidebands)

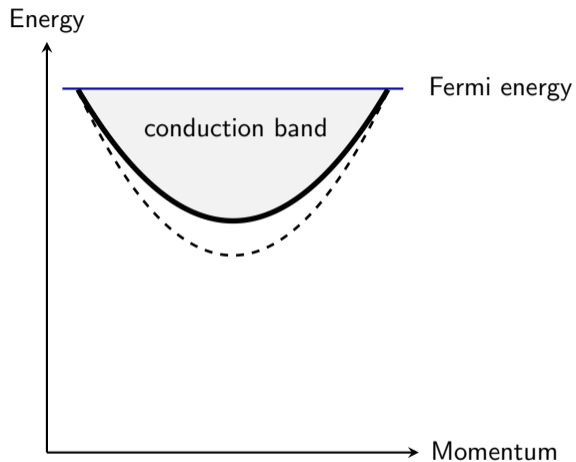


Figure adapted from Kandolf et al, Phys. Rev. B 105, 085148 (2022)



# Polaron satellites (aka phonon sidebands)

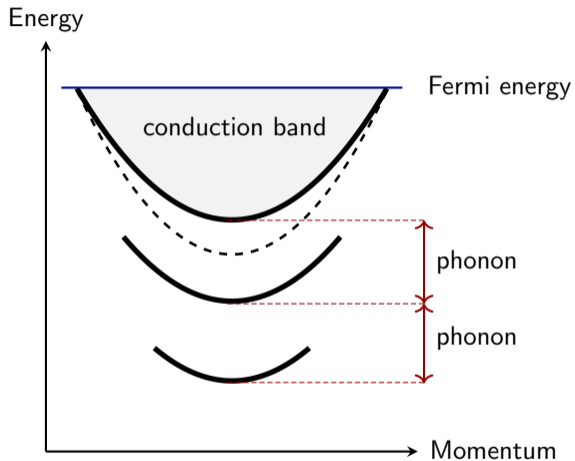


Figure adapted from Kandolf et al, Phys. Rev. B 105, 085148 (2022)

# Electron mass enhancement vs. phonon satellites in ARPES

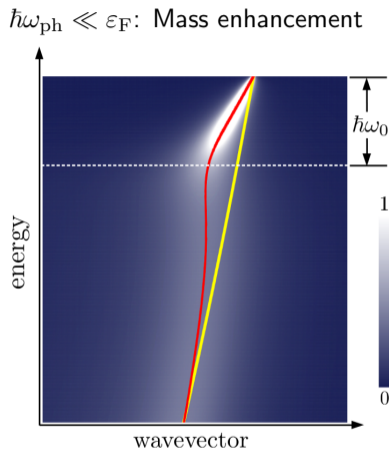
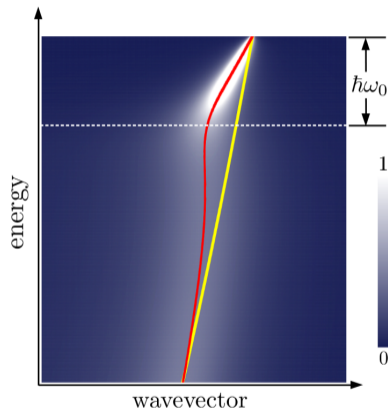


Figure from FG, Rev. Mod. Phys. 89, 015003 (2017)

# Electron mass enhancement vs. phonon satellites in ARPES

$\hbar\omega_{\text{ph}} \ll \varepsilon_{\text{F}}$ : Mass enhancement



$\hbar\omega_{\text{ph}} \sim \varepsilon_{\text{F}}$ : Satellite

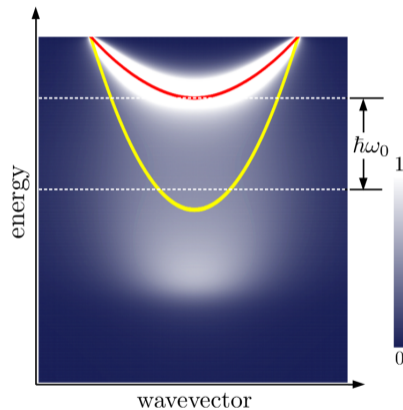


Figure from FG, Rev. Mod. Phys. 89, 015003 (2017)

# Calculated vs. measured spectral function: EuO

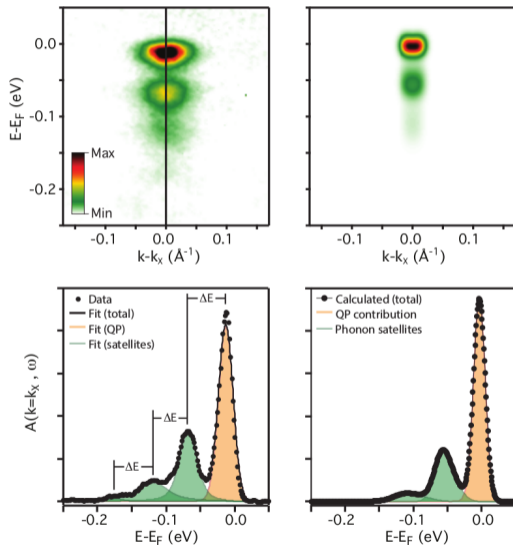
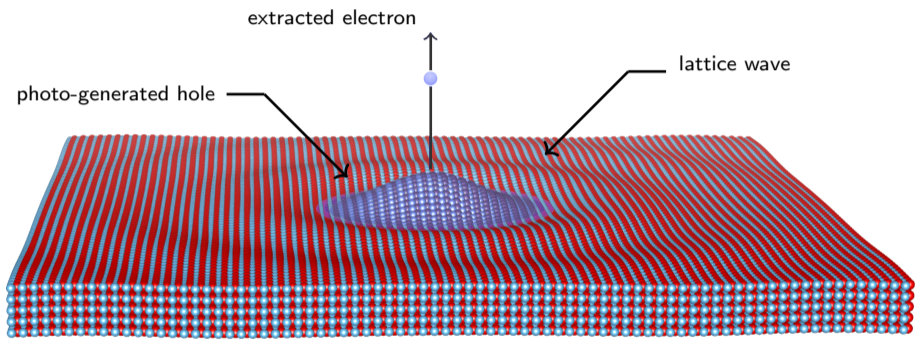


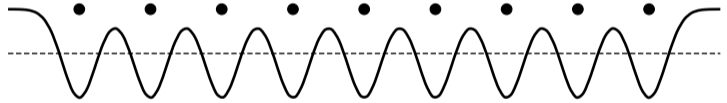
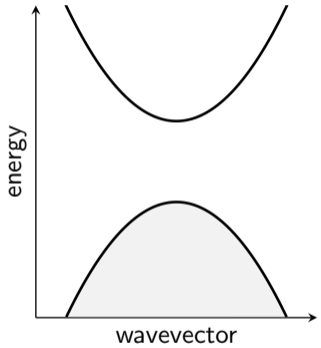
Figure from Riley et al,  
Nat. Commun. 9, 2305 (2018)

# Meaning of satellite bands

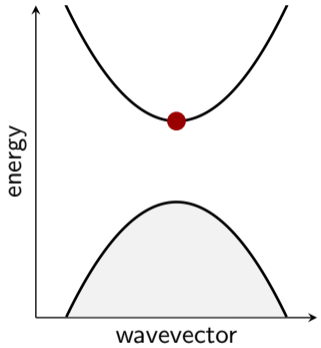


Phonon satellites are shake-up excitations

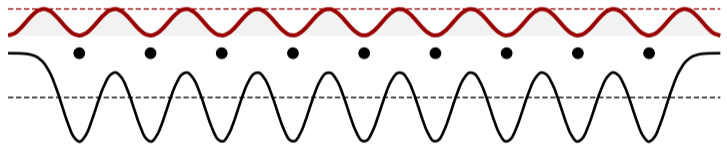
# Electrons in solids: Bloch picture vs. Landau picture



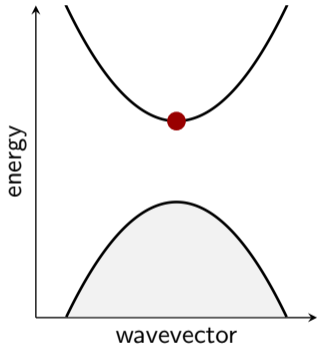
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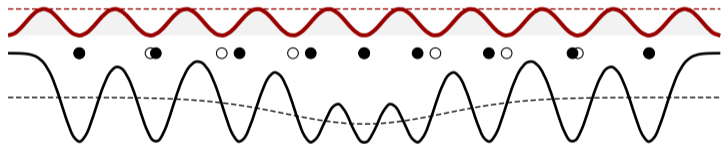
band electron **delocalized**



# Electrons in solids: Bloch picture vs. Landau picture

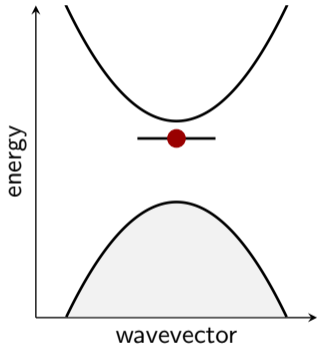


band electron **delocalized**

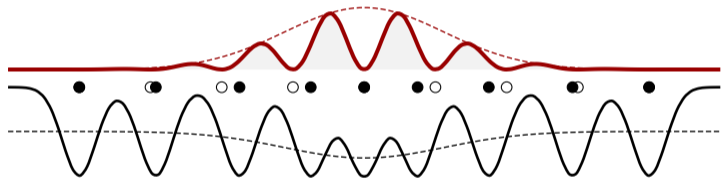




# Electrons in solids: Bloch picture vs. Landau picture



electron **localized** by lattice distortion: polaron



# Polarons in DFT calculations

Electron added to  $\text{Li}_2\text{O}_2$  ground state

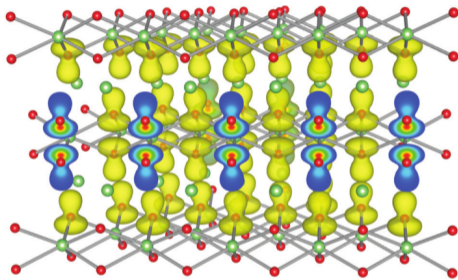
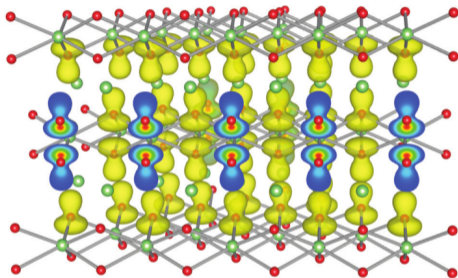


Figure from Feng et al, Phys. Rev. B 88, 184302 (2013)

# Polarons in DFT calculations

Electron added to  $\text{Li}_2\text{O}_2$  ground state



Self-localization after ionic relaxation

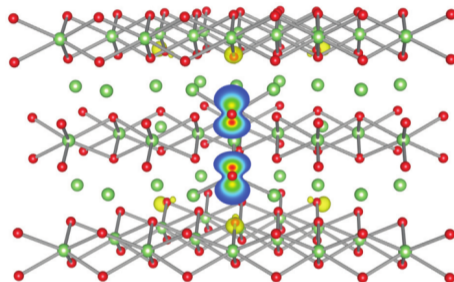
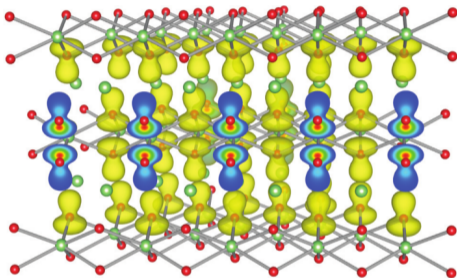


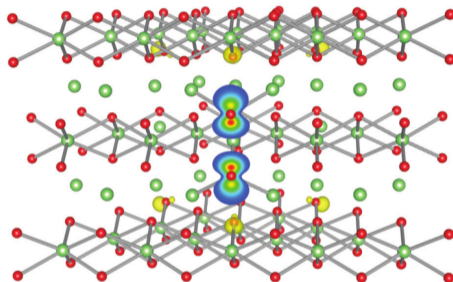
Figure from Feng et al, Phys. Rev. B 88, 184302 (2013)

# Polarons in DFT calculations

Electron added to  $\text{Li}_2\text{O}_2$  ground state



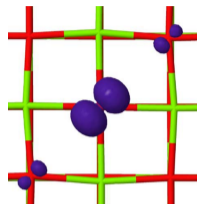
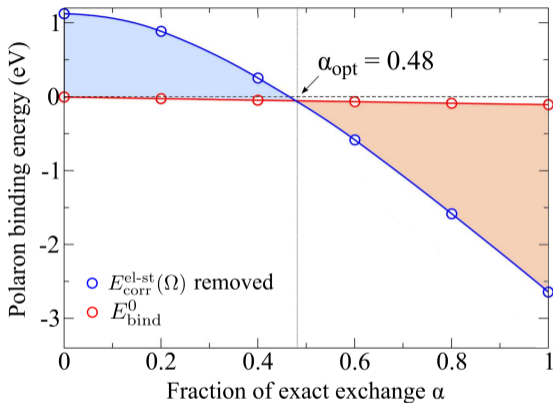
Self-localization after ionic relaxation



- Formation energy and size sensitive to the XC functional
- Only very small polarons accessible

Figure from Feng et al, Phys. Rev. B 88, 184302 (2013)

# Sensitivity to functional: hole polaron in MgO

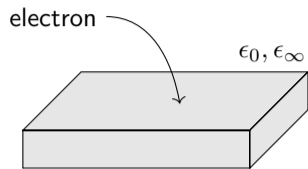


Figures adapted from Kokott, Levchenko, Rinke, Scheffler, New J. Phys. 20 (2018)

See Kokott et al for Koopman's based correction schemes

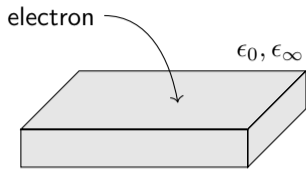
See Falletta et al, Phys. Rev. Lett. 129, 126401 (2022) for many-body self-interaction correction schemes

# Ground state of the polaron in the Landau-Pekar model



Pekar, Zh. Eksp. Teor. Fiz. 16, 341 (1946); Landau and Pekar, Zh. Eksp. Teor. Fiz. 18, 419 (1948)

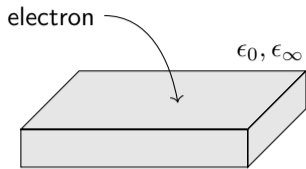
# Ground state of the polaron in the Landau-Pekar model



$$E = \frac{\hbar^2}{2m^*} \int d\mathbf{r} |\nabla\psi|^2 + \frac{1}{2} \int d\mathbf{r} \mathbf{E} \cdot \mathbf{D}$$

Pekar, Zh. Eksp. Teor. Fiz. 16, 341 (1946); Landau and Pekar, Zh. Eksp. Teor. Fiz. 18, 419 (1948)

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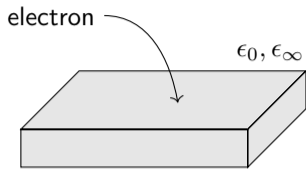
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$$\nabla \cdot \mathbf{D} = -e|\psi(\mathbf{r})|^2 \quad \mathbf{D} = \epsilon_0\epsilon_0\mathbf{E}$$

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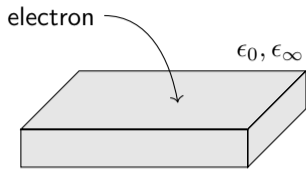
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$$\frac{1}{2} \int d\mathbf{r} \mathbf{E} \cdot \mathbf{D} = \frac{1}{2} \frac{e^2}{4\pi\epsilon_0} \left( \frac{1}{\epsilon_0} \right) \int d\mathbf{r} d\mathbf{r}' \frac{|\psi(\mathbf{r})|^2 |\psi(\mathbf{r}')|^2}{|\mathbf{r} - \mathbf{r}'|}$$

Pekar, Zh. Eksp. Teor. Fiz. 16, 341 (1946); Landau and Pekar, Zh. Eksp. Teor. Fiz. 18, 419 (1948)

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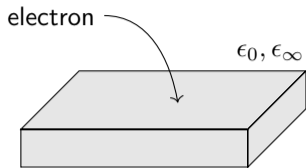
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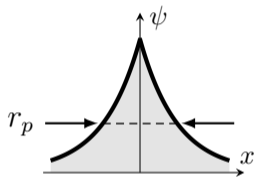
$$-\frac{\hbar^2}{2m^*} \nabla^2 \psi(\mathbf{r}) - \frac{e^2}{4\pi\epsilon_0} \left( \frac{1}{\epsilon_0} - \frac{1}{\epsilon_\infty} \right) \int d\mathbf{r}' \frac{|\psi(\mathbf{r}')|^2}{|\mathbf{r} - \mathbf{r}'|} \psi(\mathbf{r}) = \epsilon \psi(\mathbf{r})$$

Pekar, Zh. Eksp. Teor. Fiz. 16, 341 (1946); Landau and Pekar, Zh. Eksp. Teor. Fiz. 18, 419 (1948)

# Landau-Pekar equation

Simplest trial solution

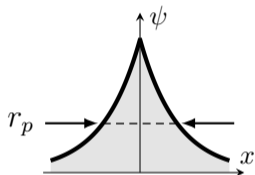
$$\psi(\mathbf{r}) = \exp(-|\mathbf{r}|/r_p)$$



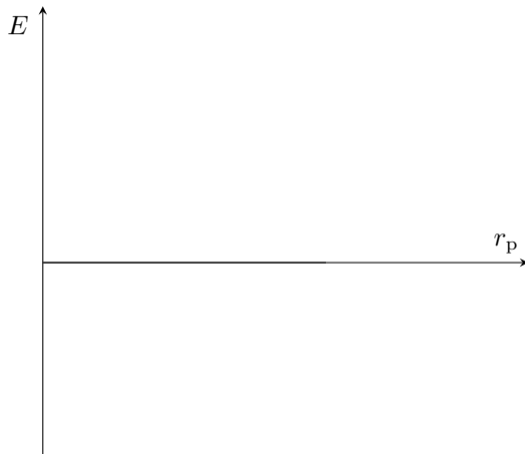
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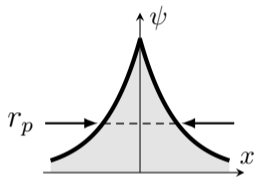
$$E =$$



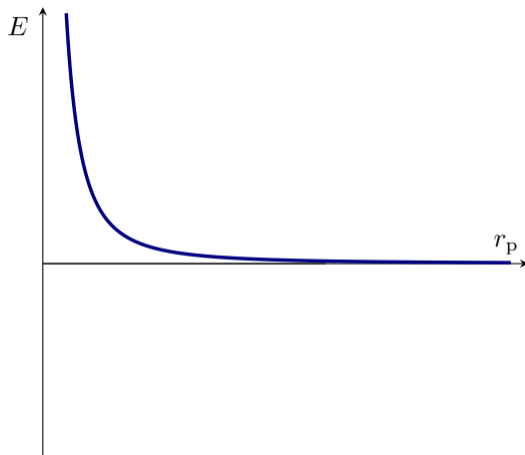
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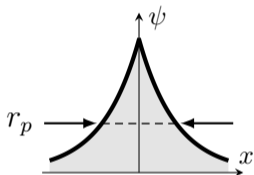
$$E = \frac{\hbar^2}{2m^*} \frac{1}{r_p^2}$$



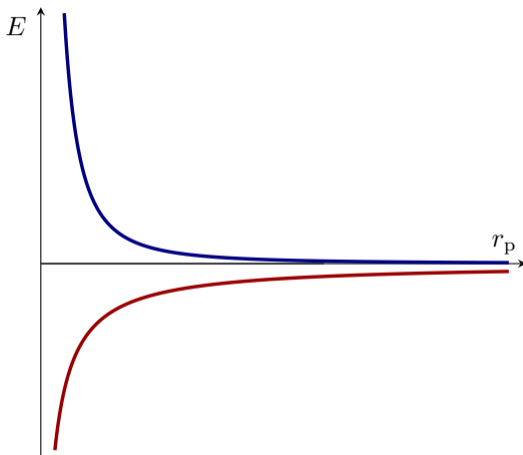
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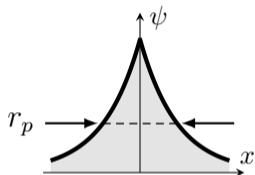
$$E = \frac{\hbar^2}{2m^*} \frac{1}{r_p^2} - \frac{5}{16} \left( \frac{1}{\epsilon_0} - \frac{1}{\epsilon_\infty} \right) \frac{e^2}{4\pi\epsilon_0} \frac{1}{r_p}$$



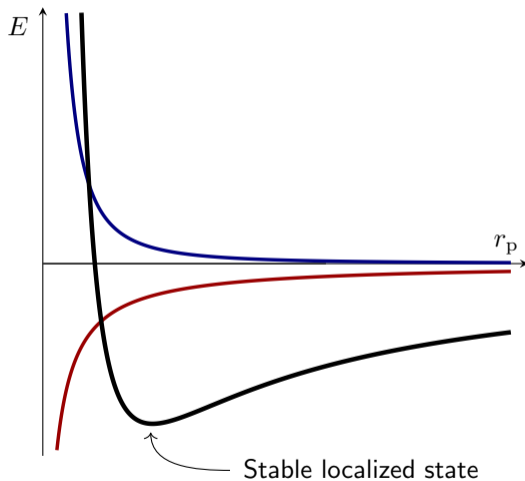
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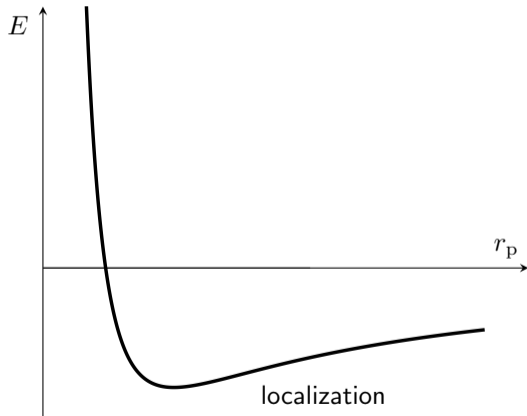
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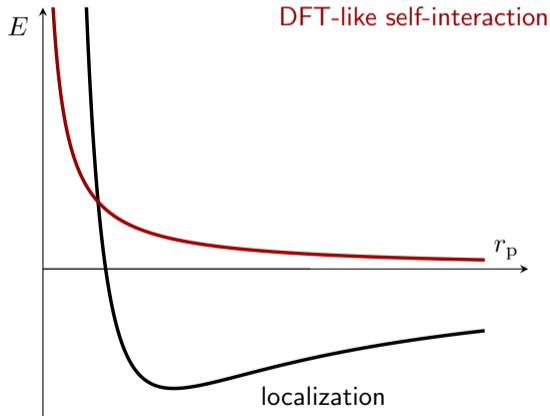
# Effect of DFT self-interaction

$$- \left( \frac{1}{\epsilon_\infty} - \frac{1}{\epsilon_0} \right) \int d\mathbf{r}' \frac{|\psi(\mathbf{r}')|^2}{|\mathbf{r} - \mathbf{r}'|}$$



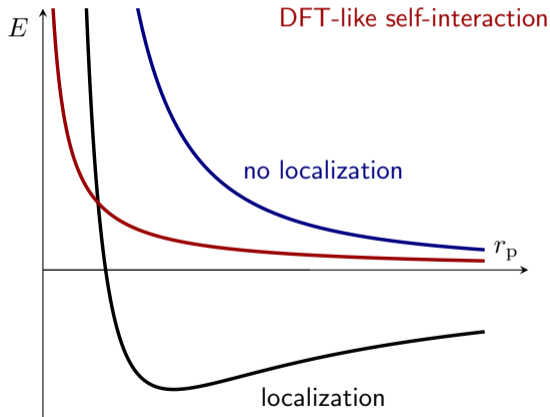
# Effect of DFT self-interaction

$$-\left(\frac{1}{\epsilon_\infty} - \frac{1}{\epsilon_0}\right) \int d\mathbf{r}' \frac{|\psi(\mathbf{r}')|^2}{|\mathbf{r} - \mathbf{r}'|} + \int d\mathbf{r}' \frac{|\psi(\mathbf{r}')|^2}{|\mathbf{r} - \mathbf{r}'|}$$



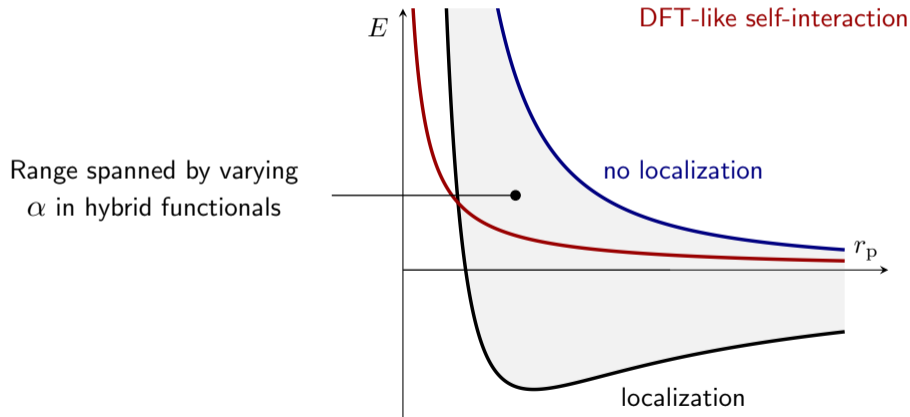
# Effect of DFT self-interaction

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# Effect of DFT self-interaction

$$-\left(\frac{1}{\epsilon_\infty} - \frac{1}{\epsilon_0}\right) \int d\mathbf{r}' \frac{|\psi(\mathbf{r}')|^2}{|\mathbf{r} - \mathbf{r}'|} + \int d\mathbf{r}' \frac{|\psi(\mathbf{r}')|^2}{|\mathbf{r} - \mathbf{r}'|}$$



$$E = \sum_{i \in \text{occ}} \int d\mathbf{r} |\nabla \psi_i|^2 + \frac{1}{2} \int d\mathbf{r} d\mathbf{r}' \frac{n(\mathbf{r})n(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} + E_{xc}[n]$$
$$+ \sum_{\kappa} \int d\mathbf{r} \frac{Z_{\kappa} n(\mathbf{r})}{|\mathbf{r} - \boldsymbol{\tau}_{\kappa}|} + \frac{1}{2} \sum_{\kappa \kappa'} \frac{Z_{\kappa} Z_{\kappa'}}{|\boldsymbol{\tau}_{\kappa} - \boldsymbol{\tau}_{\kappa'}|}$$

$$E = \sum_{i \in \text{occ}} \int d\mathbf{r} |\nabla \psi_i|^2 + \frac{1}{2} \int d\mathbf{r} d\mathbf{r}' \frac{n(\mathbf{r})n(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} + E_{xc}[n]$$
$$+ \sum_{\kappa} \int d\mathbf{r} \frac{Z_{\kappa} n(\mathbf{r})}{|\mathbf{r} - \boldsymbol{\tau}_{\kappa}|} + \frac{1}{2} \sum_{\kappa \kappa'} \frac{Z_{\kappa} Z_{\kappa'}}{|\boldsymbol{\tau}_{\kappa} - \boldsymbol{\tau}_{\kappa'}|}$$

Add one electron

$$n(\mathbf{r}) \rightarrow n(\mathbf{r}) + |\psi(\mathbf{r})|^2$$

$$\boldsymbol{\tau}_{\kappa} \rightarrow \boldsymbol{\tau}_{\kappa} + \mathbf{u}_{\kappa}$$

$$E =$$

$$E = \sum_{i \in \text{occ}} \int d\mathbf{r} |\nabla \psi_i|^2 + \int d\mathbf{r} |\nabla \psi|^2$$



$$E = \sum_{i \in \text{occ}} \int d\mathbf{r} |\nabla \psi_i|^2 + \int d\mathbf{r} |\nabla \psi|^2$$
$$+ \frac{1}{2} \int d\mathbf{r} d\mathbf{r}' \frac{[n(\mathbf{r}) + |\psi(\mathbf{r})|^2] [n(\mathbf{r}') + |\psi(\mathbf{r}')|^2]}{|\mathbf{r} - \mathbf{r}'|} + E_{xc}[n + |\psi|^2]$$

$$\begin{aligned} E &= \sum_{i \in \text{occ}} \int d\mathbf{r} |\nabla \psi_i|^2 + \int d\mathbf{r} |\nabla \psi|^2 \\ &+ \frac{1}{2} \int d\mathbf{r} d\mathbf{r}' \frac{[n(\mathbf{r}) + |\psi(\mathbf{r})|^2] [n(\mathbf{r}') + |\psi(\mathbf{r}')|^2]}{|\mathbf{r} - \mathbf{r}'|} + E_{xc}[n + |\psi|^2] \\ &+ \sum_{\kappa} \int d\mathbf{r} \frac{Z_{\kappa} [n(\mathbf{r}) + |\psi(\mathbf{r})|^2]}{|\mathbf{r} - (\boldsymbol{\tau}_{\kappa} + \mathbf{u}_{\kappa})|} + \frac{1}{2} \sum_{\kappa \kappa'} \frac{Z_{\kappa} Z_{\kappa'}}{|(\boldsymbol{\tau}_{\kappa} + \mathbf{u}_{\kappa}) - (\boldsymbol{\tau}_{\kappa'} + \mathbf{u}_{\kappa'})|} \end{aligned}$$

Formation energy functional of an extra electron, without self-interaction

$$E = \int d\mathbf{r} \psi^* \hat{H}_{\text{KS}} \psi + \int d\mathbf{r} |\psi|^2 \frac{\partial V_{\text{KS}}}{\partial \tau_{\kappa}} \cdot \mathbf{u}_{\kappa} + \frac{1}{2} \mathbf{u}_{\kappa} \cdot \mathbf{C}_{\kappa\kappa'} \cdot \mathbf{u}_{\kappa'}$$

# Polarons in density-functional perturbation theory

Formation energy functional of an extra electron, without self-interaction

$$E = \int d\mathbf{r} \psi^* \hat{H}_{\text{KS}} \psi + \int d\mathbf{r} |\psi|^2 \frac{\partial V_{\text{KS}}}{\partial \tau_{\kappa}} \cdot \mathbf{u}_{\kappa} + \frac{1}{2} \mathbf{u}_{\kappa} \cdot \mathbf{C}_{\kappa\kappa'} \cdot \mathbf{u}_{\kappa'}$$

Variational minimization with respect to  $\psi$  and  $\mathbf{u}_{\kappa}$

$$\begin{cases} \hat{H}_{\text{KS}} \psi + \psi \frac{\partial V_{\text{KS}}}{\partial \tau_{\kappa}} \cdot \mathbf{u}_{\kappa} = \lambda \psi \\ \mathbf{u}_{\kappa} = -(\mathbf{C})_{\kappa\kappa'}^{-1} \cdot \int d\mathbf{r} \frac{\partial V_{\text{KS}}}{\partial \tau_{\kappa'}} |\psi|^2 \end{cases}$$

# Polarons in reciprocal space

$$\psi(\mathbf{r}) = \frac{1}{N_p} \sum_{n\mathbf{k}} A_{n\mathbf{k}} \psi_{n\mathbf{k}}(\mathbf{r})$$
$$\mathbf{u}_\kappa(\mathbf{R}) = -\frac{2}{N_p} \sum_{\mathbf{q}\nu} B_{\mathbf{q}\nu}^* \sqrt{\frac{\hbar}{2M_\kappa\omega_{\mathbf{q}\nu}}} e^{i\mathbf{q}\cdot\mathbf{R}} \mathbf{e}_{\kappa,\mathbf{q}\nu}$$

Theory in Sio et al, Phys. Rev. Lett. 122, 246403 (2019)

# Polarons in reciprocal space

$$\psi(\mathbf{r}) = \frac{1}{N_p} \sum_{n\mathbf{k}} A_{n\mathbf{k}} \psi_{n\mathbf{k}}(\mathbf{r})$$
$$\mathbf{u}_\kappa(\mathbf{R}) = -\frac{2}{N_p} \sum_{\mathbf{q}\nu} B_{\mathbf{q}\nu}^* \sqrt{\frac{\hbar}{2M_\kappa\omega_{\mathbf{q}\nu}}} e^{i\mathbf{q}\cdot\mathbf{R}} \mathbf{e}_{\kappa,\mathbf{q}\nu}$$

$$\frac{2}{N_p} \sum_{\mathbf{q}m\nu} B_{\mathbf{q}\nu} g_{mn\nu}^*(\mathbf{k}, \mathbf{q}) A_{m\mathbf{k}+\mathbf{q}} = (\varepsilon_{n\mathbf{k}} - \varepsilon) A_{n\mathbf{k}}$$
$$B_{\mathbf{q}\nu} = \frac{1}{N_p} \sum_{m\mathbf{k}} A_{m\mathbf{k}+\mathbf{q}}^* \frac{g_{mn\nu}(\mathbf{k}, \mathbf{q})}{\hbar\omega_{\mathbf{q}\nu}} A_{n\mathbf{k}}$$

*Ab initio* polaron equations

Theory in Sio et al, Phys. Rev. Lett. 122, 246403 (2019)

# Electron polaron in LiF

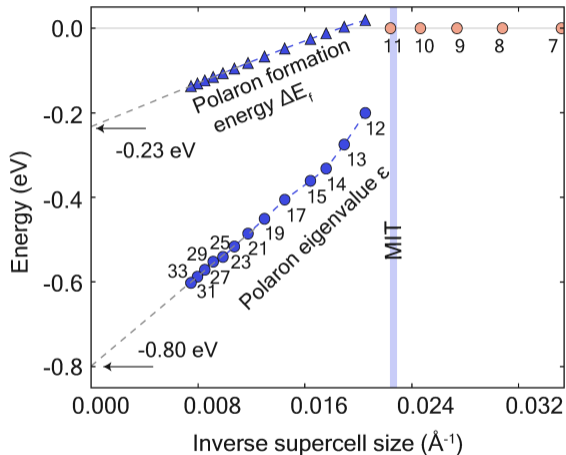


Figure from Sio et al, PRL 122, 246403 (2019)

# Electron polaron in LiF

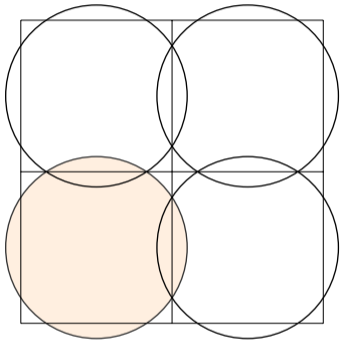
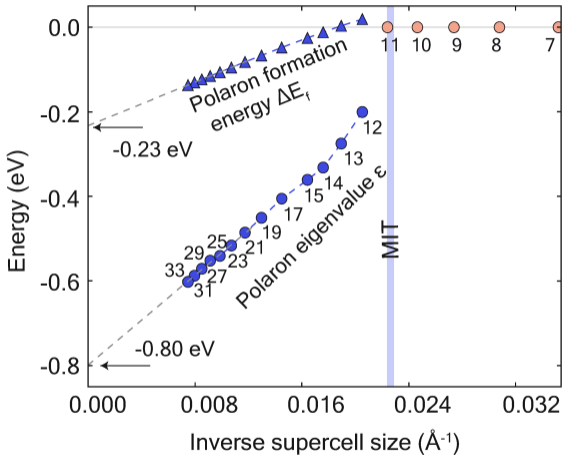


Figure from Sio et al, PRL 122, 246403 (2019)



# Electron polaron in LiF

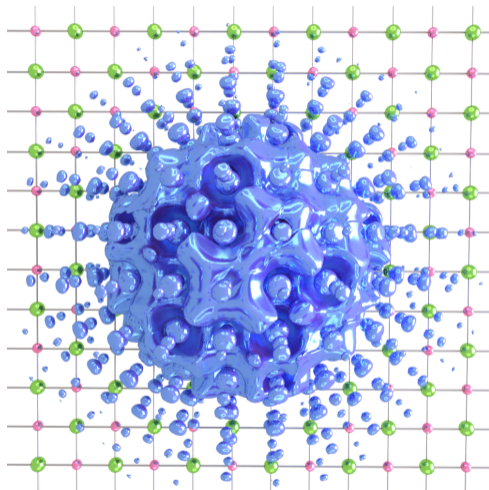
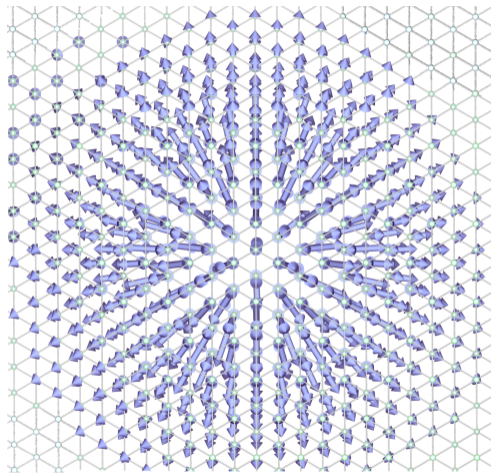


Figure from Sio et al, PRL 122, 246403 (2019)

# Electron polaron in LiF



fluorine displacements

Figure from Sio et al, PRL 122, 246403 (2019)

# Hole polaron in LiF

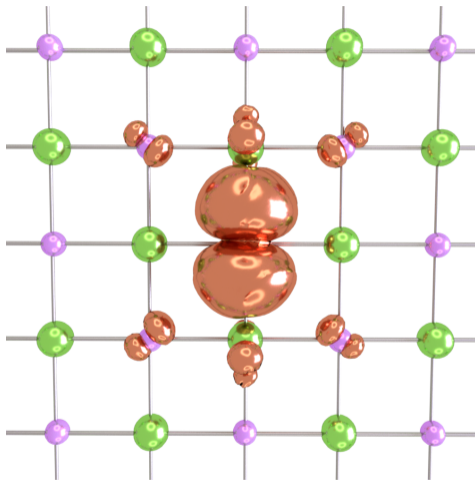
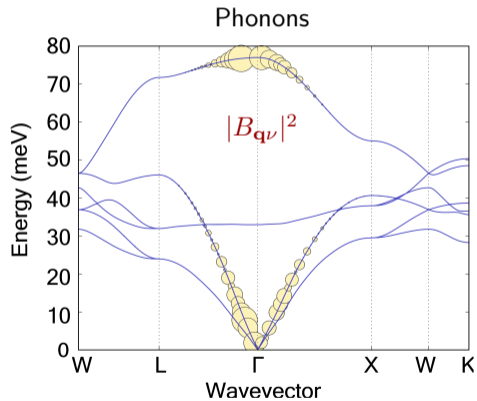
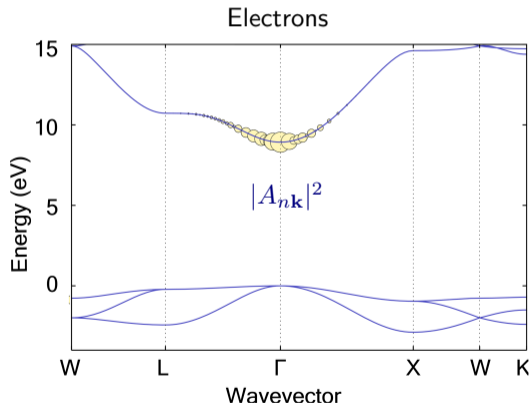


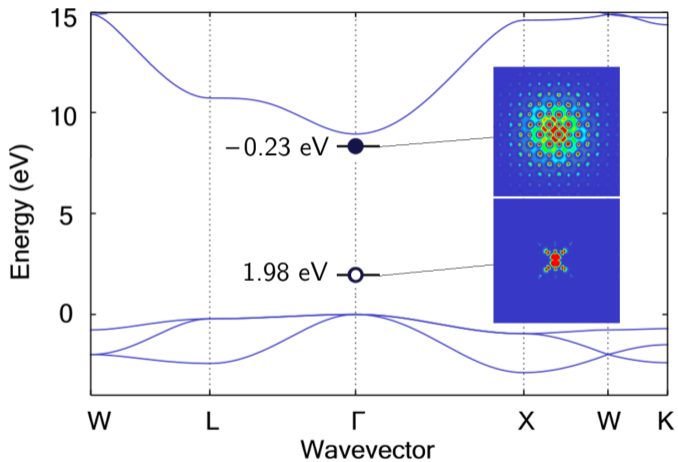
Figure from Sio et al, PRB 99, 235139 (2019)

# Polaron as coherent superposition of Bloch waves

## Electron polaron in LiF

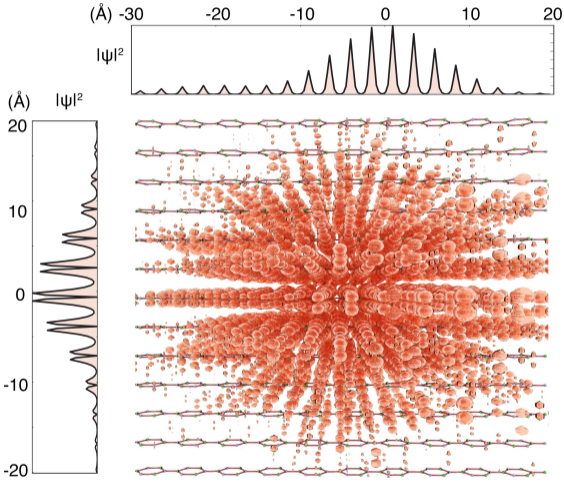


# Quasiparticle energies of polarons in LiF



Shown are formation energies w.r.t. delocalized solutions

# Hole polaron in bulk h-BN



Formation energy  
-13.6 meV

Figure from Sio et al, Nat. Phys. 19, 629 (2023)

# Hole polaron in bulk h-BN

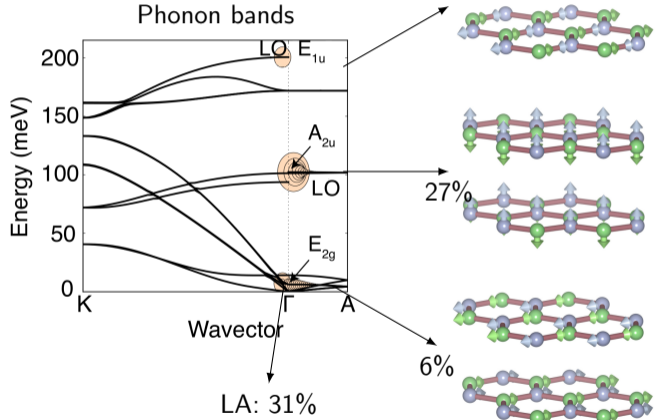
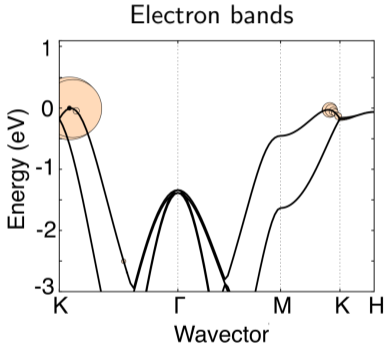
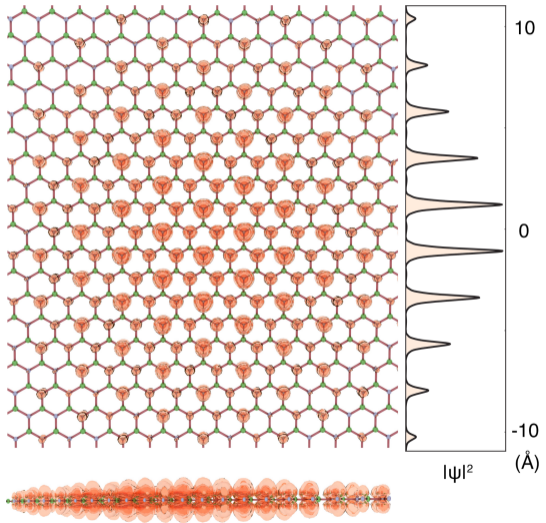


Figure from Sio et al, Nat. Phys. 19, 629 (2023)

# Hole polaron in monolayer h-BN

Formation energy  
-15.9 meV





# Hole polaron in monolayer h-BN

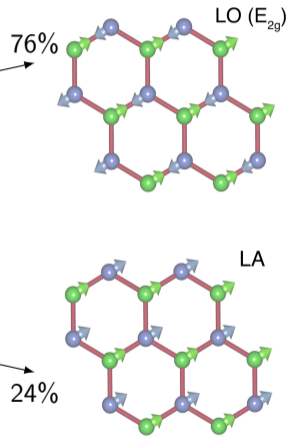
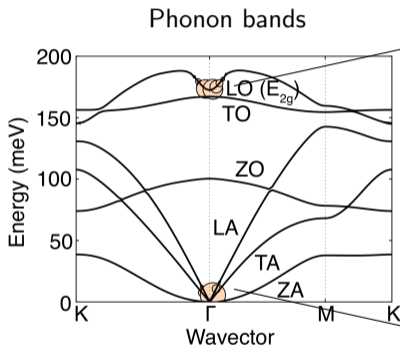
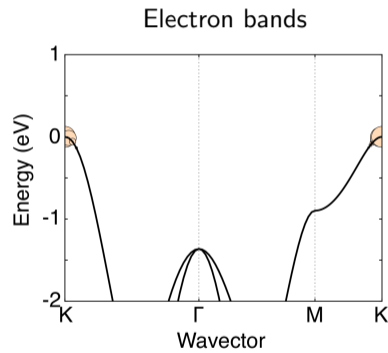


Figure from Sio et al, Nat. Phys. 19, 629 (2023)

# Many-body field-theoretic approach to polarons

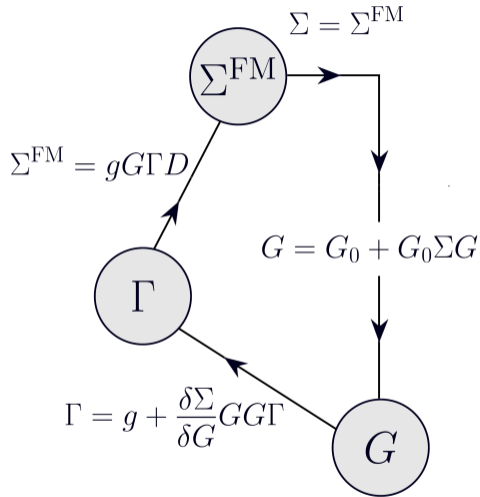


Figure from Lafuente-Bartolomé et al, Phys. Rev. Lett. 129, 076402 (2022)

# Many-body field-theoretic approach to polarons

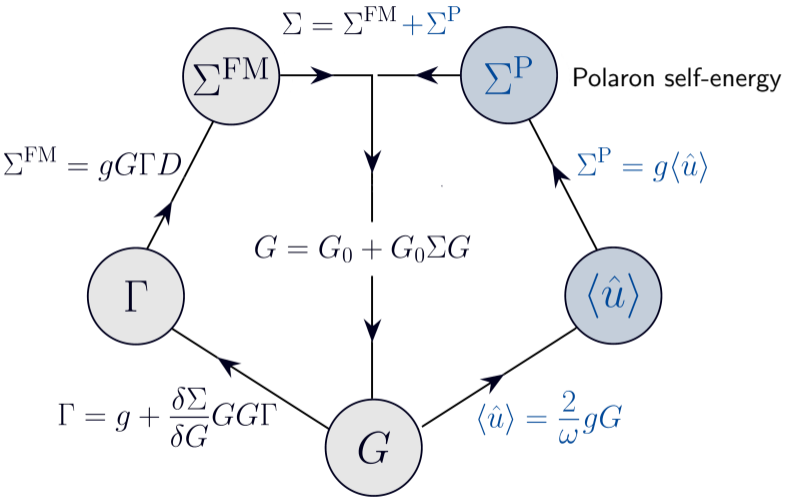


Figure from Lafuente-Bartolomé et al, Phys. Rev. Lett. 129, 076402 (2022)

# Many-body field-theoretic approach to polarons

Lehmann representation of the Green's function

$$G(\mathbf{r}, \mathbf{r}'; \omega) = \sum_s \frac{f_s(\mathbf{r}) f_s^*(\mathbf{r}')}{\hbar\omega - \varepsilon_s}$$



Dyson orbitals

$$f_s(\mathbf{r}) = \frac{1}{\sqrt{N}} \sum_{n\mathbf{k}} A_{n\mathbf{k}}^s \psi_{n\mathbf{k}}(\mathbf{r})$$

From Lafuente-Bartolomé et al, PRB 106, 075119 (2022)

# Many-body field-theoretic approach to polarons

Lehmann representation of the Green's function

$$G(\mathbf{r}, \mathbf{r}'; \omega) = \sum_s \frac{f_s(\mathbf{r}) f_s^*(\mathbf{r}')}{\hbar\omega - \varepsilon_s}$$



Dyson orbitals

$$f_s(\mathbf{r}) = \frac{1}{\sqrt{N}} \sum_{n\mathbf{k}} A_{n\mathbf{k}}^s \psi_{n\mathbf{k}}(\mathbf{r})$$

$$\sum_{n'\mathbf{k}'} \left[ \varepsilon_{n\mathbf{k}} \delta_{n\mathbf{k}, n'\mathbf{k}'} + \Sigma_{n\mathbf{k}, n'\mathbf{k}'}^{\text{FM}}(\varepsilon_s) + \Sigma_{n\mathbf{k}, n'\mathbf{k}'}^{\text{P}} \right] A_{n'\mathbf{k}'}^s = \varepsilon_s A_{n\mathbf{k}}^s$$

Many-body *ab initio* polaron equations

From Lafuente-Bartolomé et al, PRB 106, 075119 (2022)

# Comparison with Diagrammatic Monte Carlo

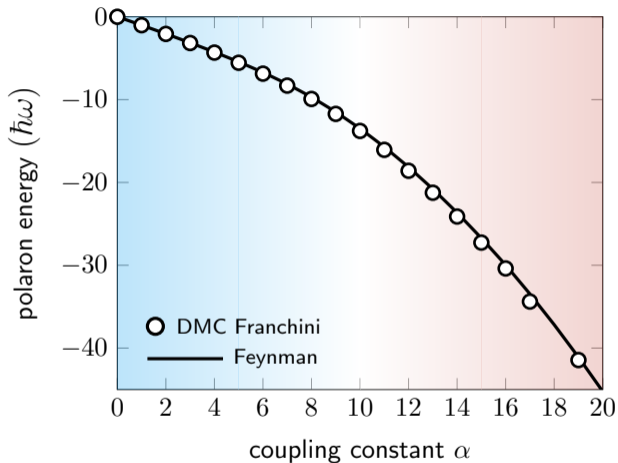


Figure from Lafuente-Bartolomé et al, Phys. Rev. Lett. 129, 076402 (2022)

Diagrammatic Monte Carlo data from: Hahn, Klimin, Tempere, Devreese, Franchini, Phys. Rev. B 97, 134305 (2018)

# Comparison with Diagrammatic Monte Carlo

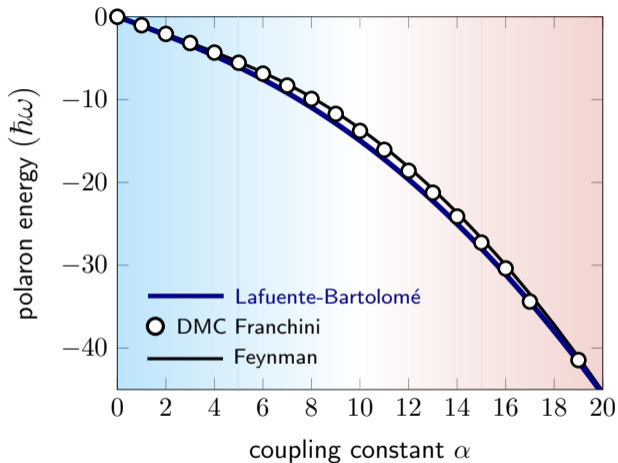
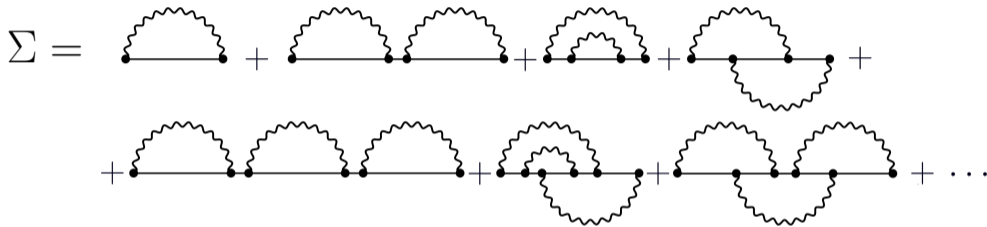


Figure from Lafuente-Bartolomé et al, Phys. Rev. Lett. 129, 076402 (2022)

Diagrammatic Monte Carlo data from: Hahn, Klimin, Tempere, Devreese, Franchini, Phys. Rev. B 97, 134305 (2018)

# Diagrammatic Monte Carlo vs. many-body polaron equations

## Diagrammatic Monte Carlo





# Diagrammatic Monte Carlo vs. many-body polaron equations

## Diagrammatic Monte Carlo

$$\Sigma = \begin{array}{c} \text{---} \text{wavy} \text{---} + \text{---} \text{wavy} \text{---} \text{wavy} \text{---} + \text{---} \text{wavy} \text{---} \text{---} \text{wavy} \text{---} + \text{---} \text{---} \text{wavy} \text{---} + \\ + \text{---} \text{wavy} \text{---} \text{wavy} \text{---} \text{wavy} \text{---} + \text{---} \text{---} \text{wavy} \text{---} \text{---} \text{wavy} \text{---} + \text{---} \text{---} \text{wavy} \text{---} \text{---} \text{wavy} \text{---} + \dots \end{array}$$

## Many-body polaron equations

$$\Sigma = \text{---} \text{wavy} \text{---} + \begin{array}{c} \text{---} \text{---} \\ | \\ \text{---} \end{array}$$

- DFT calculations of polarons suffer from the self-interaction error
- *Ab initio* polaron equations yield self-interaction-free polaron energies and wavefunctions
- These equations are the DFT approximation to a many-body Green's formalism related to the theory discussed on Monday
- There are many types of polarons, from atomic-like polarons to very large nanoscale polarons

# References

- Franchini, Reticcioli, Setvin, and Diebold, Nat. Rev. Mater. 6, 560 (2021) [\[link\]](#)
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- Lafuente-Bartolomé, Lian, Sio, Guturbay, Eiguren, and Giustino, Phys. Rev. B 106, 075119 (2022) [\[link\]](#)
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